BONUS QUESTION: HOW DOES FLEXIBLE INCENTIVE PAY AFFECT UNEMPLOYMENT DYNAMICS?

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Abstract

We introduce dynamic incentive contracts into a model of unemployment fluctuations. Our main result is that wage cyclicality from incentives does not affect the response of unemployment to productivity shocks. The response of unemployment is the same, to a first-order, in two economies: one with flexible incentive pay, and another with exogenously fixed wages. This equivalence is due to movements in effort. Under the optimal incentive contract, firms' profits do not change when wages fall, because the effort of the worker falls too.

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1 Introduction

Macroeconomists have long argued that wage rigidity is important for business cycles. If wages do not respond to business cycle shocks, then profits respond more. The consequence is large movements in unemployment (Hall, 2005; Shimer, 2005; Gertler et al., 2008; Christiano et al., 2016).

However, there is a debate about whether wages are rigid in the data. A major source of disagreement relates to incentive pay. Incentive pay, such as bonuses, is common—half of U.S. workers receive some incentive pay, including 30% of bottom-decile earners (Lemieux et al., 2009; Makridis and Gittleman, 2018). Incentive pay adjusts relatively often: bonuses change regularly for many workers, but base pay tends to be fixed (Grigsby et al., 2021). Moreover, incentive pay appears more cyclical than base pay in some studies, including for newly-hired workers (Bils, 1985; Solon et al., 1997; Swanson, 2007; Hazell and Taska, 2022; Bils et al., 2022a). How does incentive pay affect the response of the economy to shocks?

This paper investigates the behavior of unemployment in a model with dynamic incentive contracts. We consider a setting with moral hazard and persistent idiosyncratic shocks similar to Edmans et al. (2012), which we embed in a standard Diamond-Mortensen-Pissarides labor search model. When a risk-neutral firm meets a risk-averse worker, the firm offers a contract that flexibly adjusts pay in response to both aggregate and idiosyncratic productivity shocks. The optimal contract trades off incentivizing unobservable effort with insuring the worker. In the baseline model, there is no bargaining and the utility promised to workers at the start of the job is acyclical. Therefore, all fluctuations in wages are due to incentives. Wages are procyclical if firms wish to incentivize workers more during booms.

Our main result is that wage cyclicality due to incentives does not affect the response of unemployment to aggregate productivity shocks. More formally, the impulse response of labor market tightness to shocks is the same, to a first-order, in two economies. The first economy features flexible incentive pay, whereas the second economy has exogenously fixed real wages (Hall, 2005). Both economies must be calibrated to the same steady state labor share. Therefore, procyclical wages from incentives do not reduce the response of unemployment to business cycle shocks: the response of unemployment would be the same if instead wages were fixed.

This result contrasts with a well-known argument. Weitzman (1986), for instance, argues that flexible incentive pay should dampen the response of unemployment to shocks, because flexible pay reduces production costs during contractions. However Weitzman (1986) does not consider the effects of changing effort. Our model allows effort to vary, which is the key reason for our contrasting result. In labor search models, the response of profits to shocks determines vacancy posting and unemployment. With flexible incentive pay, wages fall during a contraction, which prevents profits falling too much. This effect is present in standard models. However there is a less standard *incentive effect* in our model, which acts in the opposite direction. A decline in wages weakens incentives and reduces effort, amplifying the response of profits and unemployment. Under the optimal incentive contract, the wage effect and the incentive effect cancel out exactly to a first-order, due to an envelope theorem. Therefore, profits in the flexible incentive pay economy behave as if neither wages nor effort had responded to the shock—that is, as if wages were rigid.

Our result holds with general preferences and persistent shock processes. Characterizing dynamic incentive contracts in these cases is challenging (e.g. Golosov et al., 2016). However we do not characterize the optimal contract, and instead exploit optimality using an envelope theorem. Although our equivalence result is first-order, the equivalence holds numerically for shocks of a similar size to the business cycle.[1]

Our result does not hinge on the distinction between new hires' and incumbents' wages (Pissarides, 2009). If the present value of wages for a new hire is procyclical, there is an offsetting movement in the present value of effort required by the firm.

As is well known, wage cyclicality from workers receiving more utility during booms—due to bargaining or better outside options—does dampen the response of unemployment. This form of wage cyclicality arises in standard models without incentives, which are often calibrated to match the comovement of wages and unemployment (e.g. Pissarides, 2009). If the comovement of wages and unemployment is in fact due to incentives, then standard models will understate the "effective" degree of wage rigidity.

We mention two caveats. First, our equivalence result applies to the impulse response of unemployment to business cycle shocks, which is an object of common interest in macroeconomics. For instance, the impulse response of unemployment to shocks determines the slope of the Phillips Curve for price inflation in models with nominal rigidity and search (Christiano et al., 2016). However, the response of other variables will differ between the incentive pay and rigid wage models. Notably, productivity and output dynamics will differ between the two economies because of the endogeneity of effort, evoking a notion of capacity utilization (Burnside et al., 1993). Second, our mechanism depends on effort and wages moving in the same direction over the business cycle. This pattern is consistent with the available time series evidence, but effort is hard to measure.[2]

[1] We also establish a similar result with endogenous separations (Diamond, 1982; Mortensen and Pissarides, 1994) and limited worker commitment.

[2] Various measures of worker effort—from time use surveys, variable capacity utilization, and information on workplace injuries—fall during recessions (Burda et al., 2020; Fernald, 2014; Galí and Van Rens, 2021). Further, the pass-through of idiosyncratic firm shocks to wages is procyclical (Chan et al., 2023), consistent

Related literature. A large literature has developed models consistent with the micro-evidence on price setting but tractable enough to study aggregate rigidity, often via an analytical equivalence to simpler models (e.g., Alvarez et al., 2016; Auclert et al., 2022). Other papers try to isolate which micro moments on price setting are most relevant for aggregate price rigidity (e.g., Kehoe and Midrigan, 2008; Eichenbaum et al., 2011). We aim to provide a model that is consistent with the micro-evidence on *wage* setting, while proving an equivalence to simpler models with rigid wages.

Many papers relate wage dynamics to unemployment fluctuations, and argue that wage cyclicality dampens the response of unemployment to business cycle shocks.[3] However, most papers in this literature abstract from incentive pay, which is important empirically. We provide a model that links incentive pay to unemployment dynamics.

Moen and Rosén (2011) and Zhou (2022) consider models with incentive contracts and wage posting. They find numerically that incentives amplify unemployment fluctuations. Fongoni (2024) studies a model in which wages affect effort due to exogenous reference-dependent preferences, and finds that the response of effort to wages amplifies business cycle shocks. We provide analytical results that explain how incentives affect unemployment fluctuations, and connect to simpler models with wage rigidity. Finally, Bils et al. (2022b) study a model in which incumbent workers' wages are rigid, effort is contractible, and new hires' wages are flexible. Their model produces large employment fluctuations. Instead, we study a canonical model of dynamic incentive pay with non-contractible effort and flexible wages for both incumbents and new hires, and prove an equivalence to a simple model with fixed wages and effort.

2 A Business Cycle Model with Incentive Pay

Labor market. Time is discrete. A large measure of risk-neutral firms match with workers and produce output. A unit mass of workers is either employed or unemployed and searching for a job. Let n_t denote the measure of employed workers at the start of period t, while $u_t \equiv 1 - n_t$ is the measure of unemployed workers looking for jobs. Fluctuations in labor market variables are driven by technology, which follows a first-order Markov process $\{z_t\}_{t=0}^{\infty}$ with finite lower and upper bounds $\underline{z}$ and $\overline{z}$. Denote the history of this process until t by $z^t = \{z_0, ..., z_t\}$, and denote the marginal distribution of z^t by $\hat{\pi}_t(z^t|z_0)$.

Firms post vacancies v_t to recruit unemployed workers. The number of matches made

with firms seeking to incentivize less effort during recessions.

[3] An incomplete list of papers from this vast literature includes Hall (2005); Shimer (2005); Hall and Milgrom (2008); Gertler and Trigari (2009); Elsby (2009); Rudanko (2009); Brügemann and Moscarini (2010); Kennan (2010); Gertler et al. (2020); Fukui (2020); Blanco et al. (2025) and Elsby and Gottfries (2022).

in period t is given by a constant-returns-to-scale matching function $m(u_t, v_t)$; labor market conditions are summarized by market tightness $\theta_t = v_t/u_t$, with a job finding rate $\phi(\theta_t) = m(u_t, v_t)/u_t$ and a vacancy filling rate $q_t \equiv q(\theta_t) = m(u_t, v_t)/v_t$. Let $\nu_t \equiv -d\ln q_t/d\ln\theta_t$ denote the period t elasticity of the job filling rate with respect to θ_t. Maintaining a vacancy has a per-period cost of κ. At the end of period $t-1$, an exogenous fraction s of workers separate from employment and enter unemployment. The unemployed search for jobs, so u_t evolves as

$$u_t = u_{t-1} + s(1-u_{t-1}) - \phi(\theta_{t-1})u_{t-1}(1-s). \tag{1}$$

Preferences and consumption. Workers have time-separable risk-averse preferences over consumption $c_t \in [\underline{c}, \overline{c}]$ and effort $a_t \in [\underline{a}, \overline{a}]$ and discount future payoffs by a factor $\beta \in (0,1)$. Preferences are summarized by $u(c, a)$, where u is strictly increasing and strictly concave in c, strictly decreasing and strictly convex in a, and Lipschitz continuous.

Employed workers consume their wages in each period, with newly-hired workers producing output and receiving a wage in the period in which they are hired. Unemployed workers exert no effort and are paid constant unemployment benefits b, receiving flow payoff $\xi \equiv u(b, 0)$. Therefore, the value of an unemployed worker at the start of period t is

$$U(z_t) = \phi(\theta_t)\,\mathcal{E}(z_t) + (1-\phi(\theta_t))\left(\xi + \beta\mathbb{E}\left[U(z_{t+1})\,|z_t\right]\right), \tag{2}$$

where $\mathcal{E}(z)$ is the worker's value if she begins employment when aggregate productivity is z.

Firms and vacancy posting. Firms maximize expected profits with discount factor β. Each firm employs at most a single worker. Consider a firm i that successfully matches with a worker at time 0 and starts producing in the same period. The firm's output in period t is $y_t = f(z_t, \eta_t)$, where f is strictly increasing and continuously differentiable in its arguments and η_t is an idiosyncratic shock to the firm's output that is independently distributed across firms.

At the beginning of the period, before the current value of η_t is realized, the worker exerts effort a_t that affects the distribution of idiosyncratic shocks. We assume a general process for η_t, which allows for arbitrary persistence and depends on the worker's effort. The process has lower and upper bounds $\underline{\eta}$ and $\overline{\eta}$, respectively. Define a history of idiosyncratic shocks $\eta^t = \{\eta_0, ..., \eta_t\}$. The process for η_t is a probability measure $\pi_t(\eta_t|\eta^{t-1}, a^t)$, which gives the probability of η_t being realized given the history η^{t-1} of past idiosyncratic shocks and the worker's history of actions $a^t = \{a_0, \ldots, a_t\}$. We assume worker effort affects output by shifting the distribution of η realizations continuously.

Let $J(z_0)$ denote the firm's value if it matches with a worker in some initial period $t = 0$

when aggregate productivity is known to be z_0. The firm's value from posting a vacancy at time 0 is

$$\Pi_0(z_0) \equiv q(\theta_0)J(z_0) - \kappa. \tag{3}$$

Free entry into vacancy posting guarantees that this is zero in equilibrium. We entertain two possibilities for wage setting.

Rigid wage economy. Consider a benchmark economy with exogenously rigid wages and effort as in Hall (2005). All workers' wages and effort take known constant values $w_t = \bar{w}$ and $a_t = \bar{a}$ for all firms and all t, regardless of realizations of η^t or z^t. The worker's value of employment is the utility from the match and the continuation value should the match separate:

$$\mathcal{E}(z_0) = \sum_{t=0}^{\infty} (\beta(1-s))^t \left(u(\bar{w}, \bar{a}) + \int \beta s U(z_{t+1}) \hat{\pi}_t \left(z^{t+1} | z_0\right) dz^{t+1} \right). \tag{4}$$

The firm's value of a filled vacancy is given by

$$J^{\text{rigid}}(z_0) = \sum_{t=0}^{\infty} (\beta(1-s))^t \int \left(f(z_t, \eta_t) - \bar{w}\right) \tilde{\pi}_t(\eta^t, z^t | z_0, \bar{\mathbf{a}}) d\eta^t dz^t. \tag{5}$$

The value of a filled vacancy is the expected present discounted value of production minus the rigid wage, where the expectation is taken over realizations of aggregate and idiosyncratic shocks at a fixed effort $\bar{a}$ in all dates and states.

Flexible incentive pay economy. In this economy, wages are set according to a dynamic incentive contract. The firm observes the initial value of z_0 and will later observe all realizations of aggregate shocks $\{z_t\}_{t=0}^{\infty}$. Firms additionally observe the realizations of the idiosyncratic shocks η_t in every period of the match. However, they do not observe worker effort a_t. They thus cannot observe whether an output realization is high because the worker exerted high effort or received a lucky idiosyncratic shock: a moral hazard problem.

When a firm and worker meet, the firm offers the worker a contract to incentivize effort and maximize firm value. A contract specifies a wage function mapping idiosyncratic shocks and aggregate productivity to realized wages. The contract does not condition on the worker's unobservable effort, but "recommends" an effort level given the history of aggregate and idiosyncratic shocks.

Let $(\mathbf{w}, \mathbf{a})$ denote a contract, with $\mathbf{w} \equiv \{w_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty}$ and $\mathbf{a} \equiv \{a_t(\eta^{t-1}, z^t)\}_{t=0,\eta^{t-1},z^t}^{\infty}$ so that the contract is dynamic and state contingent. Let $\mathcal{X}$ denote the space of possible contracts.

Value of a filled vacancy. Under the contract $(\mathbf{w}, \mathbf{a})$ and at initial productivity z_0, the firm's expected present value of profits from a filled vacancy is

$$V(\mathbf{w}, \mathbf{a}; z_0) = \sum_{t=0}^{\infty} (\beta(1-s))^t \int\int \left(f(z_t, \eta_t) - w_t(\eta^t, z^t)\right) \tilde{\pi}_t\left(\eta^t, z^t | z_0, \mathbf{a}\right) d\eta^t dz^t, \qquad (6)$$

where $\tilde{\pi}_t(\eta^t, z^t|\mathbf{a}) \equiv \prod_{\tau=0}^{t} \pi_\tau\left(\eta_\tau|\eta^{\tau-1}, a^\tau(\eta^{\tau-1}, z^\tau)\right) \hat{\pi}_\tau(z^\tau|z_0)$ is the probability of observing a realization of η^t and z^t given the initial z_0 and the contracted effort function $\mathbf{a}$ and $a^\tau(\eta^{\tau-1}, z^\tau)$ is the sequence of effort from periods 0 to τ.

The firm's flow profits are the difference between output and wages. The firm forms an expectation over profit realizations by integrating over the distribution of both aggregate and idiosyncratic shocks, the latter of which depends on effort. The risk-neutral firm discounts period-t profits by the economy-wide discount rate β^t and the probability $(1-s)^t$ that the match survives t periods.

The contract maximizes the value of a filled vacancy

$$J(z_0) = \max_{\{w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t)\}_{t=0,\eta^t,z^t}^{\infty} \in \mathcal{X}} V(\mathbf{w}, \mathbf{a}; z_0) \qquad (7)$$

subject to the incentive and participation constraints (IC and PC) described below.

Incentive constraint. The worker chooses effort $\tilde{\mathbf{a}} \equiv \{\tilde{a}_t(\eta^{t-1}, z^t)\}_{t=0,\eta^{t-1},z^t}^{\infty}$ to maximize utility under the contract. Therefore, the effort suggested by the firm must be incentive compatible; that is, the recommended effort $\mathbf{a}$ must be what the worker chooses given the wage schedule that the firm offers. Specifically,

$$[\mathbf{IC}]: \mathbf{a} \in \underset{\{\tilde{a}_t(\eta^{t-1}, z^t)\}_{t=0,\eta^t,z^t}^{\infty}}{\operatorname{argmax}} \sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, z^t), \tilde{a}_t(\eta^{t-1}, z^t)\Big) \tilde{\pi}_t\left(\eta^t, z^t | z_0, \tilde{\mathbf{a}}\right) d\eta^t dz^t$$
$$+ \beta s \int U(z_{t+1}) \hat{\pi}_{t+1}\left(z^{t+1}|z_0\right) dz^{t+1}\Bigg]. \qquad (8)$$

Equation (8) is the value of an employed worker at time 0, which is the sum of two terms. The first is their value conditional on the match surviving through period t, which occurs with probability $(1-s)^t$. The realized flow payoff to the worker under the contract is their utility from consuming the wage offered by the contract and providing effort, which depends on realizations of aggregate productivity z^t and idiosyncratic productivity η^t. The worker's expected utility integrates over the distribution of aggregate and idiosyncratic productivity shocks. When making effort choices, the worker trades off the disutility from higher effort with the increased probability of realizing a high output draw and thus a high wage. The second term of the worker's value is the value conditional on separation. If the contract separates in period t, the worker receives the value of unemployment at the prevailing aggregate productivity z_t. The match separates in period t with probability $(1-s)^{t-1}s$.

Participation constraint. The contract must also promise the worker a value of at least $\mathcal{E}(z_0)$, the "ex ante utility" promised by the firm to the worker at the start of the contract. The constraint is

$$[\mathbf{PC}]: \sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \tilde{\pi}_t\left(\eta^t, z^t | z_0, \mathbf{a}\right) d\eta^t dz^t \\ + \beta s \int U(z_{t+1})\,\hat{\pi}_{t+1}\left(z^{t+1}|z_0\right) dz^{t+1} \Bigg] \geq \mathcal{E}(z_0). \tag{9}$$

The left-hand side of inequality (9) is the worker's value under the contract: the objective function (8) evaluated at the effort choices suggested by the contract.[4]

Firms make take-it-or-leave-it offers to workers and promise a utility $\mathcal{E} = \mathcal{B} \equiv \xi/(1-\beta)$. Thus wages respond to shocks only due to incentives. Later, we discuss the case where workers' promised utility is cyclical, due to bargaining or outside options.

Timing. A value of z_t realizes and is common knowledge at the beginning of a period. Firms then post vacancies and matches occur. Unmatched workers remain unemployed and receive ξ. Firms offer contracts to new hires to solve (7)-(9). Workers then exert effort, after which the idiosyncratic shock η_t is realized and observed by the firm and worker. Production then occurs and the worker is paid. Finally, exogenous separations occur.

Equilibrium. Given initial unemployment u_0 and a stochastic process $\{z_t, \eta_t\}_{t=0}^{\infty}$, an equilibrium is a collection of functions $\theta(z)$, $J(z)$, $U(z)$, and $\mathcal{E}(z)$, contracts $(\mathbf{w}, \mathbf{a})(z)$, and unemployment rates u_t such that (i) the tightness $\theta(z)$ satisfies the free-entry condition (3) so $\Pi_t = 0$ for all t, (ii) unemployment u_t evolves according to equation (1), (iii) wage and effort functions $(\mathbf{w}, \mathbf{a})(z)$ solve the firm's problem (7)–(9) in the flexible incentive pay economy or $w_t = \bar{w}$ and $a_t = \bar{a}$ in the rigid wage economy, (iv) the value of unemployment $U(z)$ is given by equation (2), (v) the value of employment $\mathcal{E}(z)$ is given by equation (4) in the rigid wage economy or $\mathcal{B}$ in the flexible incentive pay economy, and (vi) the value of a filled vacancy $J(z)$ or $J^{\text{rigid}}(z)$ is given by equation (7) in the flexible incentive pay economy or equation (5) in the rigid wage economy.

[4] Our benchmark model assumes full commitment, meaning a single participation constraint at the start of the match. Online Appendix A.3 proves a similar set of results holds when workers may endogenously separate from the firm due to limited commitment.

3 Equivalence of Incentive Pay and Rigid Wages

This section explains the equivalence between flexible incentive pay and rigid wages. We first explain the intuition in an easy-to-understand static limit of our model, before stating the results in the full dynamic model.

3.1 Intuition in a Static Limit

Consider a static limit with $\beta \to 0$ and $s \to 0$, so that agents are myopic and matches last one period. Suppose further that $\eta_t = a_t + \tilde{\eta}_t$, where $\tilde{\eta}_t$ is an IID mean zero random variable so that a_t shifts the mean of the idiosyncratic shock, and the firm cannot separately observe a_t and $\tilde{\eta}_t$. Finally, we assume that the production function takes the form $y_t = z_t \eta_t = z_t(a_t + \tilde{\eta}_t)$. The firm's problem becomes

$$J^{\text{Incentive}}(z) \equiv \max_{a(z), w(z,y)} \mathbb{E}_{\tilde{\eta}}[z(a(z) + \tilde{\eta}) - w(z,y)] \tag{10}$$

$$\text{subject to} \qquad a(z) \in \arg\max_{\tilde{a}(z)} \mathbb{E}_{\tilde{\eta}}\left[u(w(z,y), \tilde{a}(z))\right] \qquad [IC]$$

$$\mathbb{E}_{\tilde{\eta}}\left[u(w(z,y), a(z))\right] \geq \mathcal{B}. \qquad [PC]$$

This notation makes explicit that wages may depend on realizations of both z and y (and thus η) but that the firm is uncertain over the realized value of $\tilde{\eta}$. Let $a^*(z)$ and $w^*(z, y)$ denote the contracted effort and wages as a function of productivity and output realizations.

As usual, this contract implies a trade-off between incentives and insurance. Absent moral hazard, firms fully insure workers against wage risk. With moral hazard, firms pass idiosyncratic shocks through to workers' wages to provide incentives. Pay is flexible, in that the firm can freely adjust wages subject to the incentive and participation constraints without further restrictions. The firm can freely vary wages with z, potentially leading to procyclical wages or effort.

Role of incentives. We now study the response of labor market tightness to shocks to aggregate productivity z. Totally log-differentiating the free entry condition (3) with respect to aggregate productivity z implies

$$\frac{d\ln\theta}{d\ln z} = \frac{1}{\nu} \cdot \frac{d\ln J}{d\ln z}. \tag{11}$$

That is, the elasticity of market tightness with respect to aggregate productivity z is proportional to the elasticity of expected profits per worker to z, where the constant of proportionality depends on the elasticity of the matching function. Moreover, the employment rate is determined by the job-finding rate $f(\theta)$, whose response is proportional to market tightness. Therefore, to understand the response of employment to aggregate productivity shocks, it is sufficient to study the response of profits per worker.

Differentiating expected profits with respect to z implies

$$\frac{dJ(z)}{dz} = \overbrace{\mathbb{E}_{\tilde{\eta}}\left[a\right]}^{\substack{\text{direct}\\ \text{productivity}}} - \overbrace{\mathbb{E}_{\tilde{\eta}}\left[\frac{dw}{dz}\right]}^{\text{wages}} + \overbrace{z\mathbb{E}_{\tilde{\eta}}\left[\frac{da}{dz}\right]}^{\text{incentives}}. \tag{12}$$

The first-order response of profits to aggregate productivity has three terms. The first is the direct productivity effect: production rises with productivity, *ceteris paribus*. The second is the standard effect of wages: wages may also increase with productivity, which lowers profits. The third term is a less standard incentive effect: effort may respond to aggregate productivity shocks. The direct productivity and wage effects are present in many models. If wages are procyclical, so dw/dz is large, then profits and employment may respond little to productivity shocks.

However, if effort increases with productivity shocks, then profits may respond strongly even if wages are procyclical. Therefore incentives can offset the effect of wages on profits, leading to large employment responses even if wages are procyclical. Wage cyclicality dampens the response of unemployment to productivity shocks only if wages move *more* than effort. So far we have not characterized the response of wages and effort. We now show that the effects of wages and effort on profits "cancel out" under the optimal incentive contract.

Equivalence between rigid wage and flexible incentive pay. We now discuss our main result in the static version of the model. To a first-order, the response of employment to shocks is the same in a flexible incentive pay economy as in an appropriately-calibrated rigid wage economy. The equivalence holds even if incentive pay is procyclical—thus incentive pay is not relevant wage flexibility for the response of employment to shocks.

First, consider equation (12) in the rigid wage economy. Both incentive and wage effects are trivially zero because neither effort nor wages respond to z. Therefore, the response of profits to labor demand shocks is just the direct productivity effect: $dJ^{\text{Rigid}}(z)/dz = \bar{a}$.

Second, consider the flexible incentive pay economy. Differentiating profits in the incentive pay economy (10) and applying the envelope theorem, we see that $dJ^{\text{Incentive}}/dz = a^*(z)$. Only the direct productivity effect remains, exactly as in the rigid wage economy.[5]

[5]The same result applies if effort (or hours) is observed and chosen by the firm without an incentive constraint.

This result holds because the wage and incentive effects are equal-sized under the optimal contract so that their effects on profits cancel out, leaving only the direct productivity effect. Although wages and effort may adjust, these fluctuations do not (to a first-order) affect the profit of a firm that is optimally choosing effort and wages. This holds even if wages are strongly procyclical under the optimal contract, so that dw/dz is large.

To gain intuition, suppose an increase in z leads the firm to encourage higher effort, because a and z are complements in production. Higher effort raises profits *ceteris paribus*. To encourage the worker to provide higher effort, the firm raises the pass-through of idiosyncratic output into wages. The worker then faces more risk, for which she must be compensated with higher average wages. Thus, wages are procyclical and flexible. Higher wages lower expected profits *ceteris paribus*.

The effects of higher effort and higher wages on profits, however, exactly cancel out. This is because the firm is indifferent at the margin between increasing expected wages and increasing worker effort under the optimal incentive contract. Expected profits respond to productivity shocks as if neither wages nor effort had changed, just as in the rigid wage economy. The response of profits—and therefore market tightness—is the same in the rigid wage and flexible incentive pay economies, as long as both economies are calibrated to have the same direct productivity effect ($\bar{a} = a^*$). Put differently, the average cost of labor on the optimal incentive contract, $\mathbb{E}[w/a]$, is rigid—even though both wages and effort may be flexible.[6]

The direct productivity effect is tied to the labor share. Rearranging dJ/dz into elasticity form, with both rigid wages and incentive pay we have

$$\frac{d\ln J}{d\ln z} = \frac{z\mathbb{E}[a]}{z\mathbb{E}[a] - \mathbb{E}[w]} = \frac{1}{1-\Lambda}$$

for $\Lambda \equiv \mathbb{E}[w]/\mathbb{E}[za]$, the labor share. Therefore, productivity shocks induce the same response of profits if both economies have the same steady state labor share.

3.2 Equivalence in the Dynamic Model

We now establish this equivalence in the dynamic model, which recognizes that employment is a long-term contract and permits general functional forms.

The main challenge for the proof is to show that an envelope theorem applies. Common general envelope theorems (e.g. Milgrom and Segal, 2002) are not well-suited for studying

[6]Our result echoes a notion of efficiency wages, whereby firms cannot lower wages without also reducing output (McDonald and Solow, 1981). However, our result arises from an optimal contracting problem with agency frictions.

problems with a continuum of nonconvex constraints. The firm's problem has this feature since there is a continuum of incentive compatibility constraints, which are not generally convex. Below, we provide sufficient conditions under which an envelope theorem can be applied to our problem.

Assumption 1. *The set Φ of feasible contracts $(\mathbf{w}, \mathbf{a}) \in \mathcal{X}$ that satisfy the incentive compatibility constraints (8) and participation constraints (9) is nonempty and compact.*

We assume nonemptiness to allow the optimal contract to exist. Online Appendix A.1.4 provides two sets of sufficient conditions under which the compactness assumption is satisfied.[7] With these assumptions, we can employ an envelope theorem which applies under a continuum of possibly non-convex constraints (Bonnans and Shapiro, 2000). Let $\mathbf{\Gamma}^*(z_0)$ denote the set of optimal contracts $(\mathbf{w}^*, \mathbf{a}^*)$ solving the firm problem (7) given z_0.

We define an "impulse response" as follows. Denote $z_t = \mathbb{E}[z_t|z_0] + \varepsilon_t$, where, by definition, ε_t is the mean-zero cumulative innovation to the process for z between 0 and t and ε_0 is 0. We study the impulse response of market tightness to changes in z_0 holding fixed ε_t for all t.

To refine some results, we will require an additional set of assumptions.

Assumption 2. *Suppose that*

(i) The production function $f(z, \eta)$ is homogeneous of degree one in z, i.e. $f(z, \eta) = z\tilde{f}(\eta)$ for some function $\tilde{f}(\cdot)$.

(ii) $\partial\mathbb{E}[z_t|z_0]/\partial z_0 = 1$ so that z_t follows a random walk

(iii) The optimal incentive contract at the nonstochastic steady state for z_t is unique.

Assumption 2-(i) is standard in many labor search models, and imposes a multiplicative relationship between idiosyncratic and aggregate shocks. Assumption 2-(ii) implies that a shock to z is permanent. This assumption appears to be a reasonable approximation to the empirical process for z_t (Shimer, 2005). Assumption 2-(iii) guarantees that there is a unique steady state labor share in the incentive pay economy.

Our main result characterizes the impulse response of labor market tightness to shocks. This impulse response is of interest in its own right in many labor search models (Ljungqvist and Sargent, 2017). Moreover the same impulse response governs the slope of the price Phillips Curve in richer labor search models with sticky prices (Christiano et al., 2016). The following theorem establishes our result.

[7] Φ is compact if either (1) the support of η_t and z_t is discrete with matches lasting at most $T < \infty$ periods, or (2) contracts are continuous and twice differentiable in their arguments (η^t, z^t), with uniformly bounded first and second derivatives and the horizon T is finite.

Theorem 1. *Maintain Assumption 1. The first-order impulse response of market tightness to a change in aggregate productivity* $d\ln z_0$ *is*

$$d\ln\theta_0 = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\frac{\partial}{\partial\ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right]d\ln z_0}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]} \tag{13}$$

in the flexible incentive pay economy, for some optimal contract $(\mathbf{w}^*,\mathbf{a}^*)$ *in* $\mathbf{\Gamma}^*(z_0)$*, and*

$$d\ln\theta_0 = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\frac{\partial}{\partial\ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\bar{\mathbf{a}}\right]d\ln z_0}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(z_t,\eta_t)-\bar{w}|z_0,\bar{\mathbf{a}}\right]}. \tag{14}$$

in a rigid wage economy with $w_t=\bar{w}$ *and* $a_t=\bar{a}$*.*

Maintain Assumptions 1 and 2. The impulse response of market tightness to z *in both economies in the neighborhood of the nonstochastic steady state for* z *is equal to*

$$d\ln\theta_0 = \frac{1}{\bar{\nu}}\left(\frac{1}{1-\Lambda}\right)d\ln z_0, \tag{15}$$

where Λ *is the steady-state labor share, defined as*

$$\Lambda \equiv \frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}[w_t|z^{SS},\mathbf{a}]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}[f(z^{SS},\eta_t)|z^{SS},\mathbf{a}]}, \tag{16}$$

expectations are evaluated in a steady state with constant aggregate productivity $z_t=z^{SS}$ *and* $\bar{\nu}$ *is the steady-state elasticity of job filling with respect to tightness.*

The proof is contained in the Appendix; as in the static model, it applies an envelope theorem to the optimal contracting problem of the firm to argue only the direct effect survives. Wage cyclicality due to incentives does not dampen the response of unemployment to shocks. Equation (13) characterizes the first-order impulse response of tightness to productivity shocks with flexible incentive pay as the direct productivity effect scaled by the present value of profits; the same is true in the rigid wage economy. Therefore, the response of tightness to shocks is the same in both economies if they feature the same direct productivity effect and the same steady state present value of profits.[8]

The final part of the theorem clarifies that, under some assumptions, economies with flexible incentive pay and rigid wages produce the same impulse response if they are calibrated to the same steady-state labor share. The labor share is the "fundamental surplus". Ljungqvist and Sargent (2017) show that the response of tightness to shocks is large when

[8]If the optimal contract is not unique, then the impulse response depends on the largest (smallest) direct productivity effect among optimal contracts when productivity increases (decreases).

the fundamental surplus is small, since the profit share is small so profits become elastic to shocks. The labor share varies for various reasons. With rigid wages, the labor share varies as the exogenous wage $\bar{w}$ changes; with flexible incentive pay, the labor share is increasing in the worker's outside option. There is no straightforward connection between the labor share and efficiency (Hosios, 1990).

The equivalence holds regardless of the behavior of wages. The present value of wages can fall after a contraction. Under the optimal contract, the present value of effort will fall too. Present value movements in wages and effort cancel out. Profits and vacancy creation are unaffected. Therefore our result holds even though with long term contracts: what matters is the present value of wages, and not the spot wage (Barro, 1977).

Our result that incentive wage flexibility does not dampen unemployment fluctuations is general. Characterizing the optimal dynamic contract is difficult because of features such as persistent idiosyncratic shocks and potentially nonseparable utility between consumption and effort (Golosov et al., 2016). Applying an envelope theorem allows us to characterize the response of profits without characterizing the optimal contract. Therefore our result holds for general production or utility functions and persistent idiosyncratic shocks.

Online Appendix A.3 proves that a version of the result also holds when separations are endogenous, because workers have limited commitment. We assume that a firm and a worker can contract on the probability that a worker will separate, given every history of productivity realizations (z^t, η^t), subject to additional incentive compatibility constraints related to separation. Since one can still apply an envelope theorem, a similar equivalence result holds.

Finally, Theorem 1 is a local result. The theorem describes the first-order response of market tightness to productivity shocks, meaning the result is exactly correct for small shocks. Online Appendix B shows that the equivalence holds numerically for larger aggregate productivity shocks of up to 5%, in a calibrated version of the incentive pay model with realistic idiosyncratic shock volatility. These shocks are of the same order as the largest business cycle shocks (Fernald, 2014).

3.3 Implications: Calibrating Wage Rigidity

Our result affects prevailing calibrations of wage rigidity. Researchers typically study labor search models without incentives or effort movements, in which wages vary only due to bargaining. One often calibrates these models to match the cyclicality of new hire wages with respect to unemployment dw/du (Pissarides, 2009). If estimates of dw/du are high, then labor costs fall during a contraction in standard models, and unemployment responds little to shocks.

Our model clarifies that dw/du is not necessarily an informative moment, since the incentive pay model can generate large unemployment responses with high dw/du. As discussed above, the response of new hire wages to productivity shocks dw/dz is high if the optimal contract requires effort and wages to respond strongly to shocks. However, du/dz will be high if the direct productivity effect is large. Thus, both wage cyclicality $dw/du = (dw/dz)/(du/dz)$ and the response of unemployment du/dz can be large.

Of course, wages can vary for reasons beyond incentive provision, such as bargaining or improving outside options. These factors affect the utility the worker receives from a match. Online Appendix A.2 extends the model to study bargaining. We allow workers' ex ante utility from the incentive contract, $\mathcal{E}(z_0)$, to increase with the shock z_0. During booms, a newly-hired worker is promised higher "ex ante utility" from the incentive contract. To fulfil this promise, the firm raises the present value of wages offered to the worker by more than effort. We show that wage cyclicality arising for these reasons dampens the response of θ to z, as in standard models. That is, if ex ante utility is more cyclical, then the response of tightness to productivity shocks is smaller. Intuitively, if the firm offers the worker higher utility during booms, then the present value of wages rises by more than the present value of effort. As a result, profits rise less during booms.

What matters for calibration is the extent to which wage cyclicality is due to bargaining or incentives. If wages vary due to bargaining in the data, then the standard model and calibration are correct. However, if wages vary due to incentives, matching dw/du with the standard model will understate unemployment responses. An important question for ongoing research is: "to what extent wage cyclicality in the data is due to bargaining or incentives?"

Online Appendix B makes one attempt at answering this question in a calibrated version of our model. We find that a significant share (47%) of wage cyclicality is due to incentives.[9] As a result, our model generates empirically reasonable unemployment and productivity dynamics, even if wage cyclicality is relatively high. To infer how incentives matter, we calibrate to estimates of passthrough into wages of idiosyncratic shocks to output and the variance of continuing workers' wage growth. Conservatively, we choose a low estimate of pass-through, which reduces the importance of incentives.[10] We conclude that one should calibrate models without incentives to wage cyclicality which is around 47% lower than the raw estimate of cyclicality.[11]

[9]Appendix B.2 discusses that empirically, bonuses appear acyclical (Grigsby et al., 2021); but other incentives, such as promotions, are cyclical (e.g. Méndez and Sepúlveda, 2012).

[10]As Appendix B.1 discusses, passthrough means firms' labor supply slopes upward (Card et al., 2018). In our model, labor supply slopes upward due to effort. If labor supply also slopes upward due to monopsony power, passthrough partly reflects rents rather than incentives, hence our conservative choice.

[11]Instead, one could calibrate wage cyclicality using the comovement between wages and output per worker

We close with a caveat. Our equivalence does not apply to the unconditional volatility of unemployment, which depends both on the impulse response of unemployment and the volatility of the shock. The volatility of productivity shocks inferred from the data will differ in models with and without incentives: with incentives, measured productivity fluctuations will include both shocks and endogenous movements due to effort (see Online Appendix B).[12]

4 Conclusion

This paper studies how incentive pay affects unemployment dynamics. We embed a dynamic principal-agent problem into a labor search model. Our main result is that wage cyclicality due to incentives does not dampen the response of unemployment to shocks. The reason is that the profits do not vary under the optimal incentive contract, since effort movements offset wage movements. A key object for calibration is the share of wage cyclicality that arises from incentives; future work measuring this share would be welcome.

dw/dp, which is less affected by incentives, because effort raises both w and p (e.g. Hagedorn and Manovskii, 2008).

[12] The volatility of unemployment will not depend on incentives per se under appropriate calibrations. Consider the static model for intuition. Equations (11) and (12) imply $Var(d\theta) \propto Var(dJ) = Var(dp) + Var(dw) - 2Cov(dp, dw)$ for $p \equiv \mathbb{E}[z(a + \eta)]$ expected output per worker. This expression does not depend on the model of incentives. Calibrating any model to have the same variance-covariance structure of output per worker p and wages w will generate the same unconditional volatility of market tightness, regardless of whether incentives affect wages and output.

Appendix: Proof of Theorem 1

We apply the envelope theorem of Bonnans and Shapiro (2000), reproduced in Online Appendix A.1, to the firm's incentive problem (7). We thus verify the theorem's conditions are satisfied by our problem. The space of individual realisations of aggregate productivities $[0, +\infty)$ is a closed subset of the normed space $(\mathbb{R}, ||\cdot||_1)$ and is therefore a Banach space. In addition, the set of feasible contracts, endowed with product topology, is non-empty and is the product of Hausdorff spaces and is therefore a non-empty Hausdorff space. The firm's objective function $V(\mathbf{w}, \mathbf{a}; z)$ and its partial Fréchet derivative are continuous and Gateaux differentiable since effort continuously affects $\pi(\eta_t|\eta^{t-1}, a^t)$.

Next, we argue that z_0 does not affect the IC and PC constraints. The only possible direct effect of z_0 on the constraints is through the probability measure since the value of unemployment is constant with take-it-or-leave-it offers. We have

$$\tilde{\pi}_t\left(\eta^t, z^t|z_0, \mathbf{a}\right) = \tilde{\pi}_t\left(\eta^t, \mathbb{E}\left[z_t|z_0\right] + \varepsilon^t|z_0, \mathbf{a}\right) = \hat{\tilde{\pi}}_t\left(\eta^t, \varepsilon^t|z_0, \mathbf{a}\right) = \hat{\tilde{\pi}}_t\left(\eta^t, \varepsilon^t|\mathbf{a}\right), \tag{17}$$

which does not directly depend on z_0. The first equality replaces z_t with the definition of an innovation $z_t = \mathbb{E}\left[z_t|z_0\right] + \varepsilon_t$. The second equality studies a new probability measure $\hat{\tilde{\pi}}_t$ defined over (η, ε), since $\mathbb{E}\left[z_t|z_0\right]$ is non-random after time 0. The third equality uses that noise η and innovations ε are uncorrelated with z_0 conditional on $\mathbf{a}$.

Finally, one must verify that there exists $M \in \mathbb{R}$ and a compact subset $C \subset \mathcal{X}$ such that for every z near z_0 the set $A(z) \equiv \{(\mathbf{w}, \mathbf{a}) \in \Phi : V\left(\mathbf{w}, \mathbf{a}; z\right) \leq M\}$ is non-empty and contained in C. Setting $C = \Phi$ and $M = \max_{z \in [\underline{z}, \overline{z}], (\mathbf{w}, \mathbf{a}) \in \Phi} V(\mathbf{w}, \mathbf{a}, z)$ verifies the condition, since Φ is compact under assumption 1, and $A(z) = C = \Phi$ because all contracts $(\mathbf{w}, \mathbf{a}) \in \Phi$ have value less than M, which is finite by the Extreme Value Theorem since $V(\cdot)$ is continuous.

Therefore, the envelope theorem of Bonnans and Shapiro (2000) applies to our problem. Applying it yields the right-hand derivative of $J(z_0)$

$$\begin{aligned} J'_+\left(z_0\right) &= \sup_{x^* \in \mathbf{\Gamma}^*(z_0)} \frac{\partial}{\partial z_0} V\left(\mathbf{w}^*, \mathbf{a}^*; z_0\right) \\ &= \sup_{x^* \in \mathbf{\Gamma}^*(z_0)} \left[\sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial z_0} \mathbb{E}\left[f\left(z_t, \eta_t\right)|\mathbf{a}^*\left(z_0\right)\right]\right], \end{aligned}$$

and the left-hand derivative

$$J'_-\left(z_0\right) = \inf_{x^* \in \mathbf{\Gamma}^*(z_0)} \left[\sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial z_0} \mathbb{E}\left[f(z_t, \eta_t)|z_0, \mathbf{a}^*\right]\right].$$

Combining the left- and right-hand derivatives, it follows that to a first-order

$$dJ\left(z_0\right) = \max_{x^* \in \mathbf{\Gamma}^*(z_0)} \left[\sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial z_0} \mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right]\right] dz_0 \tag{18}$$

where the supremum effectively converts to an infimum if the increment dz_0 is negative, and the max is attained because the space of optimal contracts is compact.

In the rigid wage economy, the change in profits per worker is trivially the direct effect:

$$\frac{dJ^{\text{rigid}}\left(z_0\right)}{dz_0} = \sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\partial}{\partial z_0} \mathbb{E}\left[f\left(z_t,\eta_t\right)|z_0,\bar{\mathbf{a}}\right]. \tag{19}$$

Multiplying and dividing equations (18) and (19) by z_0 and $J(z_0)$, then substituting into equation (11) yields (13) and (14) as desired.[13]

To prove the final part of the theorem, we assume that the left- and right-hand partial derivatives of $dJ(z_0)$ are equal.[14] We have

$$\begin{aligned}
d\ln\theta_0 &= \frac{1}{\nu_0} \frac{\sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial \ln z_0} \mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right] d\ln z_0}{\sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f(z_t,\eta_t) - w_t^*|z_0,\mathbf{a}^*\right]} \\
&= \frac{1}{\nu_0} \frac{\sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f_z\left(z_t,\eta_t\right)|\mathbf{a}^*\left(z_0\right)\right] z_0 \frac{\partial \mathbb{E}[z_t|z_0]}{\partial z_0} d\ln z_0}{\sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f(z_t,\eta_t) - w_t^*|z_0,\mathbf{a}^*\right]} \\
&= \frac{1}{\nu_0} \frac{1}{1 - \frac{\sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}[w_t^*|z^{SS},\mathbf{a}^*(z^{SS})]}{\sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}[f(z^{SS},\eta_t)|z^{SS},\mathbf{a}^*(z^{SS})]}} d\ln z_0.
\end{aligned} \tag{20}$$

The second line uses our definition of an impulse response. The third line uses the fact that $\frac{\partial \mathbb{E}[z_t|z_0]}{\partial z_0} = 1$ and $zf_z\left(z,\eta\right) = f(z,\eta)$ because $f\left(z,\eta\right)$ is homogeneous of degree one in z_t by Assumption 2. It also evaluates derivatives at the non-stochastic state in which $z_t = z^{SS}$, and divides top and bottom by expected output. This is the same as equation (15). The derivation of equation (20) for the case of rigid wages is virtually identical. □

Online Appendix A.1.5 provides an alternative proof strategy leveraging the First-Order Approach to optimal contracting.

[13]Equation (11) holds not only in the static limit but also in the full dynamic model.

[14]A sufficient condition for this to hold is that there is a unique optimal contract.

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SUPPLEMENTAL ONLINE APPENDIX; NOT FOR PRINT PUBLICATION

A Analytical Results Appendix

This appendix provides additional analytical results and discussion. Subsection A.1 provides additional details and context for the proof of Theorem 1. It additionally offers an alternate proof to the one contained in the body of the text based on the First-Order Approach. Subsection A.2 extends the model to jointly study wage cyclicality from incentives and from bargaining or outside option fluctuations. Finally, subsection A.3 extends Theorem 1 to a setting with endogenous contractible separations.

A.1 Supplemental Details for Theorem 1 Proof

This subsection contains more details for the proof of Theorem 1. It first reproduces Theorem 4.13 of Bonnans and Shapiro (2000), which is the envelope theorem invoked in the proof. Then we provide a decomposition of the response of profits per worker in the dynamic model similar to equation (12) in the static model, which provides an additional interpretation to the proof strategy. Then we rewrite the firm's problem in terms of innovations to aggregate productivity ε to give additional context to equation (17) in the proof. Next, we provide low-level sufficient conditions for the space of feasible contracts satisfying the IC and PC conditions to be compact. Finally, we provide an alternative proof environment which employs a recursive formulation of the contracting problem and the common first-order approach (FoA) employed elsewhere in the literature (Farhi and Werning, 2013).

A.1.1 Statement of Theorem 4.13 of Bonnans and Shapiro (2000)

Consider the following optimization problem:

$$\min_{x \in \mathcal{X}} V(x, z) \quad \textit{subject to } x \in \Phi$$

where z is a member of a Banach space Z, $\mathcal{X}$ is a Hausdorff topological space, $\Phi \subset \mathcal{X}$ is nonempty and closed, and $V : \mathcal{X} \times Z \to \mathbb{R}$ is continuous. Let the value function be defined as

$$J(z) \equiv \inf_{x \in \Phi(z)} V(x, z)$$

and the optimal control set be given by

$$\mathbf{\Gamma}^*(z) \equiv \arg\min_{x\in\Phi(z)} V(x,z).$$

Suppose that $z_0 \in Z$ and

1. *For all $x \in \mathcal{X}$ the function $V(x,\cdot)$ is Gateaux differentiable*
2. *$V(x,z)$ and its partial Fréchet derivative with respect to z, given by $D_z V(x,z)$, are continuous on $\mathcal{X} \times Z$*
3. *There exists $M \in \mathbb{R}$ and a compact set $C \subset \mathcal{X}$ such that for every z near z_0 the set $A(z) \equiv \{x \in \Phi : V(x,z) \leq M\}$ is non-empty and contained in C.*

Then the optimal value function $z(\cdot)$ is Fréchet directionally differentiable at z_0 and

$$J'(z_0,d) = \inf_{x\in\mathbf{\Gamma}^*(z_0)} D_z V(x,z_0)\, d,$$

where d is the direction of the Fréchet derivative and $J'(z_0,d)$ is the Fréchet derivative of J with respect to z in that direction.

This theorem provides conditions under which the total derivative of the value function with respect to some parameter z is equal to the partial derivative of the value function with respect to that parameter, taking the smallest product of the partial derivative and direction across the optimal control set. In our setting, $\mathcal{X}$ is the set of contracts, $x = (\mathbf{w}, \mathbf{a})$ and $\Phi(z)$ is the set of contracts $x \in \mathcal{X}$ that satisfy the incentive compatibility and participation constraints.

A.1.2 Decomposition of Profit Response in Dynamic Model

To study profits, we combine the IC and PC into a functional $G(\mathbf{w}, \mathbf{a})$, defined such that $G(\mathbf{w}, \mathbf{a}) \leq 0$ holds if and only if $(\mathbf{w}, \mathbf{a})$ is a feasible contract in $\mathcal{X}$ that satisfies the PC (9) and IC (8). Let $\lambda(z_0)$ denote the costate functional on these constraints, and let $\Phi \equiv \{(\mathbf{w}, \mathbf{a}) \in \mathcal{X} : G(\mathbf{w}, \mathbf{a}) \leq 0\}$ be the set of feasible contracts that satisfy the IC and PC constraints. We write the value of a filled job using the functional Kuhn–Tucker Lagrangian:[15]

$$J(z_0) = V(\mathbf{w}^*, \mathbf{a}^*; z_0) - \langle G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^* \rangle, \tag{21}$$

[15] The notation $\langle x, x^* \rangle$ denotes the value of the linear functional x^* at a point x. This notation is necessary because there is a continuum of constraints—see Section 3.1.1 of Golosov et al. (2016) for a formal definition of Lagrangians with this notation.

where the star superscripts indicate values under the optimal contract at z_0. Then, we can decompose the response of firm profits to z_0, generalizing decomposition (10) from Section 3.1. The response of profits to aggregate shocks in the flexible incentive pay economy is

$$\frac{dJ(z_0)}{dz_0} = \underbrace{\frac{\partial}{\partial z_0} V(\mathbf{w}^*, \mathbf{a}^*; z_0)}_{\text{(A) direct productivity effect on profits}} - \underbrace{\left\langle \frac{\partial}{\partial z_0} G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^*(z_0) \right\rangle}_{\text{(B) direct effect on participation and incentives}} \tag{22}$$
$$+ \underbrace{\sum_{x \in \{\mathbf{w}^*, \mathbf{a}^*\}} \left[\partial_x V(\mathbf{w}^*, \mathbf{a}^*; z_0) - \langle \partial_x G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^*(z_0) \rangle\right] \cdot \frac{dx}{dz_0} - \left\langle G(\mathbf{w}^*, \mathbf{a}^*; z_0), \frac{d\lambda^*(z_0)}{dz_0} \right\rangle}_{\text{(C) indirect effects on optimal contract and costates}},$$

where ∂_x represents the vector of partial derivatives with respect to some variable x. The direct productivity effect (A) measures how shocks to initial productivity affect the expected present value of output in all periods, where the expectation conditions on initial productivity z_0 and contracted effort $\mathbf{a}^*$.

Term (B) captures the effects on the constraints. Since z_0 affects the constraints only indirectly through the contract $(\mathbf{w}, \mathbf{a})$ in our baseline model, there is no direct effect of z_0 on constraints.

The (C) term captures the effects that the shock has on profits through changes in the firm's choice variables. (C) has three pieces. First, the shock may shift the optimal contract's wage function $\mathbf{w}^*$. This is the wage effect: the wage paid for each future realization of η^t and z^t may differ for contracts signed at different initial aggregate productivity levels z_0. Second, the shock may shift the optimal contract's recommended effort function $\mathbf{a}^*$, which affects output. This is the incentive effect. Finally, the shock may shift the value of the costates on the participation and incentive constraints.

The proof of Theorem 1 effectively shows that this (C) term is zero along the optimal contract by applying an envelope theorem. It then argues that the (B) term is also zero under the baseline model. Thus only the direct effect remains in both flexible incentive pay and rigid wage economies. As discussed in section A.2, however, bargaining and outside option fluctuations will manifest as additional (B) terms, and thus will mute profits (and so market tightness and employment) responses to exogenous productivity shocks.

A.1.3 Firm Problem in Terms of Innovations

We now write the firm problem in terms of innovations ε, following the impulse response notation introduced in the main text. Specifically, we let the contracts be given by $(\mathbf{w}, \mathbf{a}) = \{w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\}_{t=0,\eta^t,\varepsilon^t}^{\infty}$ where $w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)$ are continuous functions mapping from the history of idiosyncratic and aggregate shocks, and the initial state, to wages and effort. That is, contracts can depend on z_0 and a cumulative set of deviations from z_0. We use the fact that we consider impulse responses holding fixed a path of deviations to define the measure

$$\tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) = \prod_{\tau=0}^{t} \pi_\tau\left(\eta_\tau | \eta^{\tau-1}, a^\tau\left(\eta^{\tau-1}, \varepsilon^\tau; z_0\right), \varepsilon^\tau\right) \pi_\tau\left(\varepsilon^\tau\right),$$

where the probability measure does not depend directly on z_0 because η^t is independent of z_0 by assumption, and ε^t does not depend on z_0 by our definition of an impulse response.

Thus, the firm's problem becomes

$$J(z_0) = \max_{\mathbf{w}(z_0), \mathbf{a}(z_0)} \sum_{t=0}^{\infty} (\beta(1-s))^t \int\int \left(f\left(\mathbb{E}[z_t|z_0] + \varepsilon_t, \eta_t\right) - w_t(\eta^t, \varepsilon^t; z_0)\right) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t \tag{23}$$

subject to participation constraints

$$\sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t + \beta s \int U(\mathbb{E}[z_{t+1}|z_0] + \varepsilon_t) \hat{\pi}_t(\varepsilon^{t+1}) d\varepsilon^{t+1} \Bigg] \geq \mathcal{E}(z_0) \tag{24}$$

and incentive compatibility constraints

$$\sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), \tilde{a}_t(\eta^{t-1}, \varepsilon^t; z_0)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \tilde{\mathbf{a}}\right) d\eta^t d\varepsilon^t \Bigg] \leq \sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t |, \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t \Bigg] \tag{25}$$

for all $\tilde{\mathbf{a}} \in \mathcal{X}$.

A.1.4 Sufficient Conditions for Compactness of Φ

This section provides two sets of sufficient conditions for Φ to be compact. The first condition is that η and z have finite support, and contracts last at least T periods for T finite. The second is that contracts are continuous and twice differentiable in their arguments with uniformly bounded first and second derivatives and that T is finite.

Lemma 2. *If η and z have finite support and the time horizon is finite, then the feasible set* *of contracts Φ is compact.*

Proof. Suppose $\eta_t \in \{\eta_1, \ldots, \eta_N\}$ and $z_t \in \{z_1, \ldots, z_M\}$ have finite support and T is finite. We will show that the space of $[w, a]$ functions of (η, z) that are IC and PC is compact.

Consider a sequence of functions $[w_n, a_n]$ that are IC and satisfy the PC. The sequence $[w_n(\eta_1, z_1), a_n(\eta_1, z_1)]$ takes values in a compact set. Therefore, it has a subsequence that converges. Call it $[w_{\phi_{1,1}(n)}(\eta_1, z_1), a_{\phi_{1,1}(n)}(\eta_1, z_1)]$. Now apply the same reasoning to the sequence $[w_{\phi_{1,1}(n)}(\eta_1, z_2), a_{\phi_{1,1}(n)}(\eta_1, z_2)]$; similarly, it is in a compact set, so it has a subsequence that converges.

Through this diagonal argument, we construct $[w_{\phi_{1,1} \circ \phi_{1,2} \cdots \circ \phi_{N,M}(n)}, a_{\phi_{1,1} \circ \phi_{1,2} \cdots \circ \phi_{N,M}(n)}]$: a sub-sequence of *functions* that converges. Now we need to show that the limiting function is also in the set, that is, it is IC and satisfies PC.

The PC is a closed inequality involving continuous functions. Due to the continuity of the involved functions, the limit of any sequence of functions satisfying the PC will also satisfy it. Analogously, for the incentive compatibility constraint (IC), consider fixing an action $\tilde{a}(\cdot)$. Any sequence of $[w, a]$ satisfying the IC inequality for $\tilde{a}(\cdot)$, by the continuity of the functions involved, will satisfy it at the limit. Since this applies for all $\tilde{a}(\cdot)$, the limiting function must be IC. We began with an arbitrary sequence of $[w, a]$ that are both IC and PC, and we have shown that it has a subsequence converging to a limit that is also IC and PC. Therefore, the space of mechanisms is compact.

We can now employ a standard envelope theorem in this case, given that the choice set is compact and Corollary 4 of Milgrom and Segal (2002) applies.

□

Lemma 3. *The set of feasible contracts that satisfy the IC constraints, Φ, is compact if contracts are restricted to being continuous and twice differentiable in their arguments $\{\eta^t, z^t\}$, with uniformly bounded first and second derivatives and the horizon T is finite.*

Proof. We will show that Φ is equicontinuous.[16] Let $\Xi = [\underline{\eta}, \bar{\eta}]$ be the set of possible values for η. Consider a set of functions that are continuously differentiable on $[0, 1]$ and such that both the functions and their first and second derivatives are uniformly bounded. This means there exists some real number M such that for every function f in the set and every $x \in \Xi^T \times Z^T$, $||f(x)|| \leq M$ and for its Jacobian $||Df(x)|| \leq M$, where $|| \cdot ||$ is the Euclidian norm in $\Xi^T \times Z^T$.

Given $\epsilon > 0$, choose $\delta = \epsilon/2M$. Then for any function f in Φ and any points x and y in $\Xi^T \times Z^T$ such that $||x - y|| < \delta$, by the mean value theorem, we have $||f(x) - f(y)||_\infty =$

[16] Φ is said to be equicontinuous at a point $x \in \Xi^T \times Z^T$ if, for every $\epsilon > 0$, there exists a $\delta > 0$ such that for every function f in Φ and every point y in $\Xi^T \times Z^T$, if $||x - y|| < \delta$ then $||f(x) - f(y)||_{C^1} < \epsilon$, where $|| \cdot ||_{C^1}$ is the C^1 norm.

$|Df(c)| \cdot ||x-y||$ for some c in the line $xt + (1-t)y$, $t \in [0;1]$. Since $|Df(c)| \leq M$ and $|x-y| < \delta = \epsilon/2M$, we get $||f(x) - f(y)||_\infty < \epsilon/2$.

Similarly, we can apply the mean value theorem to the Jacobian of f, and since the second derivatives are bounded, an analogous argument to that above yields $||Df(x) - Df(y)||_\infty < \epsilon/2$. Therefore $||f(x) - f(y)||_{C^1} \equiv ||f(x) - f(y)||_\infty + ||Df(x) - Df(y)||_\infty < \epsilon$ and we have shown that Φ is equicontinuous. By the Ascoli Theorem, any sequence in Φ thus has a subsequence that converges. Therefore, Φ is compact. □

A.1.5 Proof Using First-Order Approach and Recursive Formulation

We now pursue the First-Order Approach (FoA) to prove Theorem 1. To make progress, we must introduce one additional assumption guaranteeing that the so-called FoA offers a valid solution to the contracting problem:

Assumption 3. *The set of feasible contracts* $(\mathbf{w}, \mathbf{a}) \in \mathcal{X}$ *is compact and convex. Assume standard Inada conditions on utility,* $\lim_{c \to \underline{w}} u_c(c,a) = \lim_{a \to \bar{a}} u_c(c,a) = \infty$ *and* $\lim_{c \to \bar{w}} u_c(c,a) = \lim_{a \to \underline{a}} u_c(c,a) = 0$*, that the worker's optimal effort choices are determined by the first-order condition to problem* (8) *and that the density of* η_t *can be expressed as*

$$\pi_t \left(\eta_t | \eta^{t-1}, a^t \right) = \pi_t \left(\eta_t | \eta_{t-1}, a_t \right).$$

Under this assumption, the incentive compatibility constraint may be written as the first-order condition to the worker's problem, and the firm's contracting problem may be expressed recursively.

We now show how to apply an envelope theorem to the flexible incentive pay problem under this stronger assumption. This proof is clarifying because the approach is closer to standard practice, and it will also be useful because it derives results that are necessary for Proposition 4. Therefore, for this subsection, we make both Assumption 1 from the main text and 3.

The application of the envelope theorem proceeds in three steps in this environment. First, we derive a first-order approach to simplify incentive constraints into local incentive constraints as in Farhi and Werning (2013) or Pavan et al. (2014). Then, we develop a recursive formulation of the problem following the steps in Ndiaye (2020). Finally, we use these constructions to prove our main theorem.

Step 1: First-Order Approach The first-order condition for a_t in the worker's problem (8) given a contract is

$$0 = \int \int \left[u_a \Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \tilde{\pi}_t \left(\eta^t, z^t | z_0, \mathbf{a} \right) + u \Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \frac{\partial}{\partial a_t} \tilde{\pi}_t \left(\eta^t, z^t | z_0, \mathbf{a} \right) \right] d\eta^t dz^t$$

Note that this holds for every t and realization of z^t. Thus, one can remove the outer integral to write first-order incentive constraints as

$$\int \left[u_a \Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \pi_t \left(\eta_t | z^t, \eta^{t-1}, \mathbf{a} \right) + u \Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \Big) \frac{\partial}{\partial a_t} \pi_t \left(\eta_t | z^t, \eta^{t-1}, \mathbf{a} \right) \right] d\eta_t = 0$$

Step 2: Recursive Formulation We will work with the relaxed problem and develop a recursive formulation of the firm's problem. Notationally, let the value of some variable X in the period t problem be given by X, the value of X in $t-1$ be given as X_-, and the value of X in $t+1$ be given by X'. Suppressing explicit dependence of the problem on initial productivity z_0 for notational convenience, the recursive formulation of the firm's problem is then (we now drop the history dependence with the assumption that the process for η is a Markov process):

$$\begin{aligned} J(v_-, \eta_-, z_-, t) = \max_{a(\eta_-, z), w(\eta, z), v(\eta, z)} \int \int \Big[& f(\eta, z) - w(\eta, z) \\ & + \beta (1-s) J(v(\eta, z), \eta, z, t+1) \Big] \pi (\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z|z_-) d\eta dz \end{aligned} \tag{26}$$

subject to the following constraints:

$$\omega(\eta, z) = \quad u (w(\eta, z), a(\eta_-, z)) + \beta \left[(1-s) v(\eta, z) + s \textstyle\int U(z') \hat{\pi}(z'|z) dz' \right] \tag{27}$$

for all η and z realizations,

$$[\lambda]: \quad v_- \leq \int \int \omega(\eta, z) \pi (\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z|z_-) d\eta dz, \tag{28}$$

and the first-order incentive constraints:

$$\int \left[u_a \Big(w(\eta, z), a(\eta_-, z) \Big) \pi (\eta | z, \eta_-, a) + u \Big(w(\eta, z), a(\eta_-, z) \Big) \frac{\partial}{\partial a} \pi (\eta | z, \eta_-, a) \right] d\eta = 0. \tag{29}$$

The firm begins period t knowing the prior realization of shocks z_- and η_- and inherits a utility it must promise to the worker over the remaining life of the contract, which we denote v_-. The firm's flow profits are the expected output $f(\eta, z)$ minus their expected wage payments $w(\eta, z)$. Firms additionally receive a continuation value with probability $1-s$, which they discount at rate β. The firm maximizes the sum of flow profits and continuation values by choosing the suggested effort and wage functions for every realization of η and z, as well as a function for the next period's promised utility to the worker $v(\eta, z)$, subject to some constraints that we now describe.

The worker's value under the contract given a realization (η, z) is given by $\omega(\eta, z)$, defined in equation (27). It is equal to the worker's flow utility $u(w(\eta, z), a(\eta_-, z)$ plus a continuation value. With probability s, the match dissolves, and the worker receives the value of unemployment. With probability $1-s$, the match survives, and the worker receives $v(\eta, z)$.

The recursive version of the participation constraint states that the worker's expected value under the contract must be at least the value promised to them v, and is given by equation (28). Note that v_- in the initial period of the match maps to the utility promised to the worker overall $\mathcal{B}$ in the non-recursive formulation of the problem. For periods after the start of the contract, equation (28) may be interpreted as a promise-keeping constraint. Equation (29) is the relaxed incentive constraint described above.

Let the Lagrangian of the recursive problem be defined by $\int\int \mathcal{L}(\cdot)d\eta dz$ for

$$\begin{aligned}\mathcal{L} \equiv &[f(\eta, z) - w(\eta, z; z_0)]\pi(\eta|z, \eta_-, a(\eta_-, z))\,\hat{\pi}(z|z_-) \\ &+ \beta(1-s)\Big[J(v(\eta, z), \eta, z, t+1)\Big]\pi(\eta|z, \eta_-, a(\eta_-, z))\,\hat{\pi}(z|z_-) \\ &- \lambda[v_- - \omega(\eta, z)\pi(\eta|z, \eta_-, a(\eta_-, z))\,\hat{\pi}(z|z_-)] \\ &- \gamma(z)\left[u_a\Big(w(\eta, z), a(\eta_-, z)\Big)\pi(\eta|z, \eta_-, a) + u\Big(w(\eta, z), a(\eta_-, z)\Big)\frac{\partial}{\partial a}\pi(\eta|z, \eta_-, a)\right],\end{aligned} \tag{30}$$

where λ is the Lagrange multiplier on the participation constraint and $\gamma(z)$ is the multiplier on the incentive constraint given aggregate productivity z. Again, we suppress dependence on z_0, but the firm's choice variables and the distribution of z and η may all depend on z_0.

Next, we introduce the change of variable with the notation $z_t = \mathbb{E}[z_t|z_0] + \varepsilon_t$, where by definition, ε_t is the cumulative innovation to the process for z between 0 and t and ε_0 is known to be 0. One can write the Lagrangian as:

$$\begin{aligned}\mathcal{L} = &[f(\eta, \mathbb{E}[z|z_0] + \varepsilon) - w(\eta, \varepsilon)]\pi(\eta|\varepsilon, \eta_-, a(\eta_-, \varepsilon))\,\hat{\pi}(\varepsilon|\varepsilon_-) \\ &+ \beta(1-s)\Big[J(v(\eta, \varepsilon), \eta, \varepsilon, t+1)\Big]\pi(\eta|\varepsilon, \eta_-, a(\eta_-, \varepsilon))\,\hat{\pi}(\varepsilon|\varepsilon_-) \\ &- \lambda[v_- - \omega(\eta, \varepsilon)\pi(\eta|\varepsilon, \eta_-, a(\eta_-, \varepsilon))\,\hat{\pi}(\varepsilon|\varepsilon_-)] \\ &- \gamma(\varepsilon)\left[u_a\Big(w(\eta, \varepsilon), a(\eta_-, \varepsilon)\Big)\pi(\eta|\varepsilon, \eta_-, a) + u\Big(w(\eta, \varepsilon), a(\eta_-, \varepsilon)\Big)\frac{\partial}{\partial a}\pi(\eta|\varepsilon, \eta_-, a)\right]\end{aligned} \tag{31}$$

Step 3: Envelope Theorem We seek to apply Theorem 1 of Marimon and Werner (2021), which relies on the following technical assumptions.

Technical Assumptions:

TA1. The set $\mathcal{X}$ of feasible allocations is convex, and f, u, π, u_a, and π_a are continuous functions of $\{a, w, z_0\}$

TA2. The constraint set $\mathcal{G}(z_0) = \{(\mathbf{w}, \mathbf{a}) \in \mathcal{X} : G(\mathbf{w}, \mathbf{a}; z_0) \leq 0\}$ is compact for every $z \in Z$, a neighborhood of z_0, and there exists a contract $(\mathbf{w}, \mathbf{a})$ such that the participation constraint (28) is slack.

TA3. The set of optimal contracts is non-empty.

We argue these conditions apply in our setting. $\mathcal{X}$ is convex as the product of segments. Under Assumption 1, $\mathcal{X}$ is compact. Then the constraint set $\mathcal{G}(z_0)$ is a closed subset of a compact and so is compact. What's more, there exists a contract such that the participation constraint is slack since, for every z_0 and promised utility v_-, there exists a feasible continuation value and effort $\omega(\eta, z), a(\eta_-, z)$ that yield strictly higher utility than v_-: that is inequality (28) is strict. Finally, since from Assumption 1, $\mathcal{X}$ is compact and non-empty, the set of optimizers of our continuous objective (i.e., the set of optimal contracts) is non-empty.[17]

One can now apply the envelope theorem of Marimon and Werner (2021) to argue that the derivative of the value function with respect to all variables the firm chooses and costates – a^*, w^*, v^*, λ^*, and γ^* – sum to zero. Therefore, differentiating the Lagrangian (31) with respect to z_0 and substituting in for $\omega(\eta, \varepsilon)$ yields the right-hand derivative:

$$\frac{\partial J(v_-, \eta_-, z_-, t)}{\partial z_0^+} = \sup_{(\mathbf{w}^*, \mathbf{a}^*) \in \mathbf{\Gamma}^*(z_0)} \int \int \frac{\partial}{\partial z_0} [f\left(\eta, \mathbb{E}[z|z_0] + \varepsilon\right)] \pi\left(\eta|\varepsilon, \eta_-, a^*(\eta_-, \varepsilon)\right) \hat{\pi}(\varepsilon|\varepsilon_-) d\eta d\varepsilon \tag{32}$$

$$+ \beta\left(1 - s\right) \int \int \frac{\partial}{\partial z_0^+} \Big[J\left(v^*(\eta, \varepsilon), \eta, \varepsilon, t + 1\right)\Big] \pi\left(\eta|\varepsilon, \eta_-, a^*\right) \hat{\pi}(\varepsilon|\varepsilon_-) d\eta d\varepsilon$$

$$+ \beta s \lambda^*(\eta_-, z_-) \int \int \frac{\partial}{\partial z_0} U\left(\mathbb{E}[z'|z_0] + \varepsilon'\right) \hat{\pi}\left(\varepsilon'|\varepsilon\right) \hat{\pi}(\varepsilon|\varepsilon_-) d\varepsilon' d\varepsilon.$$

This is a refinement of a recursive version of equation (22): the first-order impact of aggregate productivity on the value of a filled job is given by the sum of the direct effect on the firm's flow and continuation values, plus the direct effect on the constraints. Two terms are missing

[17] This envelope theorem is better suited for our purposes than Corollary 5 of Milgrom and Segal (2002) since it does not require compactness assumptions on the support of the shocks.

from the fuller decomposition in equation (22). First, the "B-term" features no direct effect on incentive constraints. This arises from the assumption that the distribution of η and ε do not directly depend on z_0. Second, the "C-term" – the indirect effect on firm value that arises from changes in the contracted wages or effort – does not appear because we have applied the envelope theorem of Marimon and Werner (2021).

We can write explicitly the sequence of participation constraints from time 0 as:

$$\begin{aligned} \lambda_-(z_0): &\qquad \mathcal{E}(z_0) \leq v \\ \lambda_{t-1}(\eta^{t-1}, z^{t-1}): &\qquad v_{t-1}(\eta^{t-1}, z^{t-1}) \leq \int\int \omega(\eta^t, z^t)\pi\left(\eta_t | z^t, \eta^{t-1}, a(\eta^{t-1}, z^t)\right)\hat{\pi}(z_t|z^{t-1}) d\eta_t dz_t,\ \forall t \geq 1. \end{aligned}$$

The corresponding sequential participation constraints are:

$$\begin{aligned} &[\lambda_-(z_0)]: \quad \mathcal{B}(z_0) \leq \sum_{t=0}^{\infty} (\beta(1-s))^t \Big[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t \\ &+ \beta s \int U\left(\mathbb{E}[z_{t+1}|z_0] + \varepsilon_{t+1}\right) \hat{\pi}_{t+1}\left(\varepsilon^{t+1}\right) d\varepsilon^{t+1} \Big] \\ &[\lambda_\tau(\eta^\tau, \varepsilon^\tau; z_0)]: \quad \sum_{t=\tau+1}^{\infty} (\beta(1-s))^{t-\tau-1} \Big[\int\int u\Big(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\Big) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t \\ &+ \beta s \int U\left(\mathbb{E}[z_{t+1}|z_0] + \varepsilon_{t+1}\right) \hat{\pi}_{t+1}\left(\varepsilon^{t+1}\right) d\varepsilon^{t+1} \Big] \geq v_\tau(\eta^\tau, \varepsilon^\tau; z_0), \quad \forall \tau = 0, \ldots, +\infty \end{aligned} \tag{33}$$

Now we apply the envelope theorem to the problem recursively, replacing $\mathcal{E}$ with its equilibrium value $\mathcal{B}$ to obtain

$$\begin{aligned} \frac{\partial J}{\partial z_0} &= \sup_{\{w^*, a^*\} \in \mathbf{\Gamma}^*(z_0)} \sum_{t=0}^{+\infty} \Big[\int\int (\beta(1-s))^t \frac{\partial f\left(\mathbb{E}[z_t|z_0] + \varepsilon_t, \eta_t\right)}{\partial z_0} \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}(z_0)\right) d\eta^t d\varepsilon^t \qquad (34) \\ &- \lambda_-(z_0) \Big[\frac{\partial \mathcal{B}(z_0)}{\partial z_0} - \beta s \sum_{t=1}^{+\infty} \int\int (\beta(1-s))^{t-1} \frac{\partial U\left(\mathbb{E}[z_t|z_0] + \varepsilon_t\right)}{\partial z_0} \hat{\pi}\left(\varepsilon^t\right) d\varepsilon^t \Big] \\ &+ \sum_{\tau=0}^{\infty} \int\int \lambda_\tau(\eta^\tau, \varepsilon^\tau; z_0) \beta s \Big[\sum_{t=\tau+2}^{+\infty} \int\int (\beta(1-s))^{t-\tau-2} \frac{\partial U\left(\mathbb{E}[z_t|z_0] + \varepsilon_t\right)}{\partial z_0} \hat{\pi}\left(\varepsilon^t | \varepsilon^\tau\right) d\varepsilon^t \Big] \times \\ &\tilde{\pi}_\tau\left(\eta^\tau, \varepsilon^\tau | \mathbf{a}(z_0)\right) d\eta^\tau d\varepsilon^\tau \end{aligned}$$

When the outside option of the worker is acyclical and TIOLI, we have:

$$\frac{\partial J(z_0)}{\partial z_0} = \sup_{\{w^*,a^*\}\in\mathbf{\Gamma}^*(z_0)} \sum_{t=0}^{\infty} (\beta(1-s))^t \left[\int\int \frac{\partial f(\mathbb{E}[z_t|z_0]+\varepsilon_t,\eta_t)}{\partial z_0} \tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}^*(z_0)\right) d\eta^t d\varepsilon^t\right]. \tag{35}$$

The preceding equation is the same as equation (18) from the Appendix, the previous application of the envelope theorem. Therefore, the same manipulations performed at the end of the Appendix under Assumption 2 yield the market tightness dynamics in Theorem 1. □

A.2 Non-incentive reasons for wage cyclicality

To derive Theorem 1, we assumed that workers' promised utility is acyclical. This assumption provides the clean benchmark in which all wage cyclicality in the model arises due to incentives. We now relax this assumption to consider non-incentive reasons for wage cyclicality. To do so, we assume that the ex ante utility promised to the worker $\mathcal{E}(z_0)$ is given by a reduced-form function $\mathcal{B}(z_0)$: firms commit to providing workers with utility $\mathcal{B}(z_0)$ over the contract.[18] Common bargaining protocols in the labor search literature implicitly define different functions for $\mathcal{B}(z_0)$. For instance, if firms make take-it-or-leave-it offers to workers, the value of employment is equal to the value of nonemployment: $\mathcal{B}(z_0) = \sum_t \beta^t \mathbb{E}[\xi(z_t)|z_0]$, where $\xi(z_t)$ is the flow value of unemployment. This nests the case in which unemployment benefits or the opportunity cost of unemployment is procyclical (Hagedorn et al., 2013; Chodorow-Reich and Karabarbounis, 2016; Mitman and Rabinovich, 2019). Nash bargaining also implicitly defines an increasing function for $\mathcal{B}(z_0)$, as we prove below. Our formulation also evokes a notion of unemployment as a "worker discipline device" (Shapiro and Stiglitz, 1984): if the value of employment is low because unemployment at present or in the future is costly, workers will offer higher effort at lower wages.

The reduced-form approach has two advantages. First, our conclusions about the role of bargaining and outside options will be robust to a specific protocol. Second, we can tractably incorporate bargaining into dynamic incentive contract models. The disadvantage of the approach is that $\mathcal{B}(z_0)$ is a reduced-form object, which is not invariant to changes in the primitives of the environment.

Let $\mathcal{W}(z_0)$ denote the present discounted value of wage payments under the optimal wage contract:

$$\mathcal{W}(z_0) \equiv \sum_{t=0}^{\infty} (\beta(1-s))^t \int w_t^*(\eta^t, z^t)\tilde{\pi}_t(\eta^t, z^t|z_0, \mathbf{a}^*(z_0)) d\eta^t dz^t.$$

[18] See Blanchard and Galí (2010) and Michaillat (2012) for an implementation of this approach in search models without effort.

Likewise, Let $\mathcal{Y}(\mathbf{a}^*(z_0), z_0)$ denote the expected present discounted value of output from a match given the optimal effort function $\mathbf{a}^*(z_0)$:

$$\mathcal{Y}(\mathbf{a}^*(z_0), z_0) \equiv \sum_{t=0}^{\infty}(\beta(1-s))^t \int\int f(z_t, \eta_t)\tilde{\pi}_t(\eta^t, z^t|z_0, \mathbf{a}^*(z_0))d\eta^t dz^t.$$

Denote as $\tilde{\mathcal{B}}(z)$ the ex ante utility promised to the worker at the start of the contract, net of her continuation value with regard to separation into unemployment: $\tilde{\mathcal{B}}(z) \equiv \mathcal{B}(z) - \sum_{t=0}^{\infty}(\beta(1-s))^t\beta s\mathbb{E}[U(z_{t+1})|z_0 = z]$. Fluctuations in $\tilde{\mathcal{B}}(z)$ capture variations in workers' ex ante utility due to, for instance, bargaining power or changes in worker outside options. Finally, define *non-incentive wage cyclicality (NWC)* as wage movements over and above output movements driven by effort fluctuations

$$\frac{\partial\mathcal{W}^{\text{non-incentive}}(z_0)}{\partial z_0} \equiv \frac{d\mathcal{W}(z_0)}{dz_0} - \partial_{\mathbf{a}}\mathcal{Y}\left(\mathbf{a}^*(z_0); z_0\right)\frac{d\mathbf{a}^*}{dz_0}. \tag{36}$$

Note that Theorem 1 states that non-incentive wage cyclicality is zero under Assumption 1 when $\tilde{B}(z_0)$ is acyclical; this is the sense in which $\frac{\partial\mathcal{W}^{\text{non-incentive}}(z_0)}{\partial z_0}$ is not related to incentives.

Characterizing the response of market tightness to productivity becomes more difficult when $\tilde{\mathcal{B}}(z)$ is nonconstant, as the set of contracts satisfying the participation constraint now moves directly with z_0. We therefore must maintain Assumption 3 in Appendix Section A.1.5 so that the FoA offers a valid solution to the contracting problem. This then yields the following proposition:

Proposition 4. *Assume that Assumption 1 hold. The impulse response of market tightness to aggregate shocks in the flexible incentive pay economy is*

$$d\ln\theta_0 = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}(\beta(1-s))^t\frac{\partial}{\partial\ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*(z_0)\right] - \frac{\partial\mathcal{W}^{non\text{-}incentive}(z_0)}{\partial\ln z_0}}{\sum_{t=0}^{\infty}(\beta(1-s))^t\mathbb{E}\left[f(z_t,\eta_t) - w_t^*(z_0)|z_0,\mathbf{a}^*(z_0)\right]}d\ln z_0, \tag{37}$$

where $\partial W^{non-incentive}(z_0)/\partial\ln z_0$ is defined in equation (36). Moreover, if Assumption 3 additionally holds, then

$$\frac{\partial\mathcal{W}^{non\text{-}incentive}(z_0)}{\partial\ln z_0} > 0 \quad\Longleftrightarrow\quad \tilde{\mathcal{B}}'(z_0) > 0;$$

i.e., Non-Incentive Wage Cyclicality is positive if and only if ex ante utility is procyclical.

Equation (37) shows that Non-Incentive Wage Cyclicality (NWC) appears alongside the direct productivity effect. Therefore, when NWC is high, the impulse response of tightness is small – wage cyclicality arising for non-incentive reasons dampen employment responses to shocks as in the standard model. The proposition also shows that what we have defined as NWC corresponds to the cyclicality of workers' ex ante utility—NWC is positive if and only if the utility promised to workers at the start of a contract is procyclical.

A.2.1 Proof of Proposition 4

First, we derive equation (37). Differentiating the value of a filled job $J(z_0)$ with respect to z_0 yields the following expression:

$$\frac{dJ\left(z_0\right)}{dz_0}=\frac{\partial\mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)}{\partial z_0}-\left(\frac{d\mathcal{W}\left(z_0\right)}{dz_0}-\partial_{\mathbf{a}}\mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)\frac{d\mathbf{a}^*}{dz_0}\right). \tag{38}$$

and from equation (36) we have

$$\frac{\partial\mathcal{W}^{\text{non-incentive}}\left(z_0\right)}{\partial z_0}\equiv\frac{d\mathcal{W}\left(z_0\right)}{dz_0}-\partial_{\mathbf{a}}\mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)\frac{d\mathbf{a}^*}{dz_0}.$$

The preceding two equations imply

$$\begin{aligned}\frac{dJ\left(z_0\right)}{dz_0}&=\frac{\partial\mathcal{Y}\left(\mathbf{a}^*\left(z_0\right);z_0\right)}{\partial z_0}-\frac{\partial\mathcal{W}^{\text{non-incentive}}\left(z_0\right)}{\partial z_0}\\&=\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\frac{\partial}{\partial z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right]-\frac{\partial\mathcal{W}^{\text{non-incentive}}\left(z_0\right)}{\partial z_0}\end{aligned}$$

$$\begin{aligned}\implies\quad\frac{d\ln J\left(z_0\right)}{d\ln z_0}&=\frac{z_0\left(\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\frac{\partial}{\partial z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right]-\frac{\partial\mathcal{W}^{\text{non-incentive}}(z_0)}{\partial z_0}\right)}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]}\\&=\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\frac{\partial}{\partial\ln z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right]-\frac{\partial\mathcal{W}^{\text{non-incentive}}(z_0)}{\partial\ln z_0}}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]},\end{aligned}$$

which is equation (37).

We now prove the "moreover" statement that NWC is positive if and only if the promised utility is procyclical. The derivation makes use of equation (34) derived in Section A.1.5. Suppose the optimal contract features optimal choices for wages and effort, which are in the interior of Φ, which is true under the Inada conditions made in Assumption 3. Then the additional Lagrangian terms after time zero are non-binding, and equation (34) becomes

$$\frac{\partial J}{\partial z_0^+} = \sup_{\{w^*,a^*\}\in\mathbf{\Gamma}^*(z_0)} \sum_{t=0}^{\infty} \Big[\int\int (\beta(1-s))^t \frac{\partial f(\mathbb{E}[z_t|z_0]+\varepsilon_t,\eta_t)}{\partial z_0} \tilde{\pi}_t\left(\eta^t,\varepsilon^t|\mathbf{a}^*(z_0)\right) d\eta^t d\varepsilon^t \tag{39}$$

$$-\lambda_{PC}^*(z_0)\Big[\frac{\partial \mathcal{B}(z_0)}{\partial z_0} - \beta s \sum_{t=1}^{+\infty} \int\int (\beta(1-s))^{t-1} \frac{\partial U(\mathbb{E}[z_t|z_0]+\varepsilon_t)}{\partial z_0} \hat{\pi}\left(\varepsilon^t\right) d\varepsilon^t \Big]$$

Under Assumption 1, $\mathcal{X}$ is compact, and so the supremum is achieved at a contract $\{w^*, a^*\} \in \mathbf{\Gamma}^*(z_0)$. Evaluated at that optimum and comparing equation (37) with equation (39) yields

$$\lambda_{PC}^*(z_0)\left[\frac{\partial \tilde{\mathcal{B}}(z_0)}{\partial z_0}\right] = \frac{\partial \mathcal{W}^{non-incentive}(z_0)}{\partial z_0}.$$

Finally, $\lambda_{PC}^*(z_0) > 0$ because the participation constraint must bind on the optimal contract. It immediately follows that ${\partial \mathcal{W}^{non-incentive}(z_0)}/{\partial z_0} > 0$ if and only if ${\partial \tilde{\mathcal{B}}(z_0)}/{\partial z_0} > 0$. $\square$

A.2.2 Implicit Definition of $\mathcal{B}(z_0)$ under Nash Bargaining

This subsection shows that Nash bargaining implicitly defines a functional form for $\mathcal{B}(z_0)$. Suppose that the firm and worker engage in generalized Nash bargaining over the surplus of the match, and φ is the firm's bargaining power. Firms and workers take as given the utility that workers would receive were they to match with another firm next period $\mathcal{E}(z)$. Promised utility $\mathcal{B}(z_0)$ is implicitly defined by

$$\mathcal{B}(z_0) = \arg\max_{\overline{\mathcal{B}}} J\left(z_0, \overline{\mathcal{B}}\right)^{\varphi} \left(\overline{\mathcal{B}} - U(z_0)\right)^{1-\varphi}.$$

Here, as in the main text, $U(z_0)$ is the value of unemployment at time 0. $J\left(z_0, \overline{\mathcal{B}}\right)$ is defined by equations (7)-(9) in the main text, replacing $\mathcal{E}(z_0)$ with $\overline{\mathcal{B}}$ in equation (9). Therefore, $\mathcal{B}(z_0)$ is the solution to the standard Nash bargaining problem, albeit in an environment with dynamic incentive pay. The solution is

$$\varphi \frac{\frac{\partial J(z_0,\mathcal{B}(z_0))}{\partial \overline{\mathcal{B}}}}{J(z_0,\mathcal{B}(z_0))} + \frac{(1-\varphi)}{\mathcal{B}(z_0) - U(z_0)} = 0. \tag{40}$$

Note that when a firm and worker bargain, they take the expected outcome of a worker bargaining with other firms as given. Thus $U(z_0)$ does not itself depend directly on $\mathcal{B}(z_0)$. Therefore, equation (40) implicitly characterizes a particular choice for $\mathcal{B}(z_0)$ from the Nash bargain.

A.3 Endogenous Separations and Limited Worker Commitment

This section introduces efficient endogenous separations and limited worker commitment into the baseline environment. To economize, we only discuss the parts of the model that change due to efficient separations or limited worker commitment; otherwise, the model is the same as the flexible incentive pay economy of the main text.

A.3.1 Economic Environment

Labor Market As in the baseline model of the main text, a large measure of risk-neutral firms match with unemployed workers according to a frictional matching technology. Fluctuations are driven by aggregate productivity z_t, and there is free entry to vacancy posting at a constant flow cost κ, as in the main text.

At the end of period $t-1$ an endogenous fraction s_t of workers separate from employment and enter unemployment. The unemployed search for new jobs, so u_t evolves as

$$u_t = u_{t-1} + s_t(1 - u_{t-1}) - \phi(\theta_{t-1})u_{t-1}(1 - s_t). \tag{41}$$

Preferences and Consumption Workers' preferences are identical to the model of the main text.

Firms and Wage Setting The value of a firm of posting a vacancy at time 0 is

$$\Pi_0 = q(\theta_0)\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\prod_{j=1}^{t} (1 - s_j)\right) (f(z_t, \eta_t) - w_t)\right] - \kappa, \tag{42}$$

where $\mathbb{E}$ conditions on the firm's information set at time 0 prior to meeting a worker. A vacancy is filled with probability $q(\theta)$. If a firm meets a worker, its value is the expected present value of the difference between production and wage payments, discounted by the firm's discount factor β as well as separation risk. Here, $\prod_{j=1}^{t}(1 - s_j)$ is the endogenous probability that a match survives until period t, which cumulates the probability $1 - s_j$ that a match survives period j. We entertain two possibilities for wage setting.

Flexible Incentive Pay Economy As in the main text, the firm observes realizations of both z_t and η_t, but does not observe worker's effort. When a firm and a worker meet, the firm offers the worker a contract to incentivize effort and maximize firm value. The innovation of this section is that the firm now has the additional option to vary the probability that the match separates in each date and state. For instance, if the expected present value of profits has turned negative, the firm may choose to terminate the contract. Thus the contract may be summarized by functions $w_t(\eta^t, z^t) \in [\underline{w}, \bar{w}]$, $a_t(\eta^{t-1}, z^t) \in [\underline{a}, \bar{a}]$ and a separation probability $s_t(\eta^t, z^t) \in [0,1]$ for all t and all realizations of η^t and z^t. Let $(\mathbf{w}, \mathbf{a}, \mathbf{s})$ denote a contract, with $\mathbf{w} \equiv \{w_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty}$, $\mathbf{a} \equiv \{a_t(\eta^{t-1}, z^t)\}_{t=0,\eta^{t-1},z^t}^{\infty}$ and $\mathbf{s} \equiv \{s_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty}$. Let $\mathcal{X}$ denote the space of possible contracts.

Value of a Filled Vacancy. Under the contract $(\mathbf{w}, \mathbf{a}, \mathbf{s})$, and initial productivity z_0, the firm's expected present value of profits from posting a vacancy is

$$V(\mathbf{w}, \mathbf{a}, \mathbf{s}; z_0) = \sum_{t=0}^{\infty} \int \int \beta^t \mathcal{S}_t\left(\eta^t, z^t\right) \left(f(z_t, \eta_t) - w_t(\eta^t, z^t)\right) \tilde{\pi}_t\left(\eta^t, z^t | z_0, \mathbf{a}\right) d\eta^t dz^t, \tag{43}$$

where $\mathcal{S}_t\left(\eta^t, z^t\right) \equiv \prod_{j=1}^{t} \left(1 - s_{t-j}\left(\eta^{t-j}, z^{t-j}\right)\right)$ is the probability that a match survives the sequence η^t, z^t; and $\tilde{\pi}_t(\eta^t, z^t | z_0, \mathbf{a})$ is the probability of observing a realization of η^t and z^t given the initial z_0 and the contracted effort function $\mathbf{a}$, as in the main text. The risk-neutral firm discounts period t profits by the economy-wide discount rate β^t and the probability $\mathcal{S}_t(\eta^t, z^t)$ that the match survives t periods.

The contract maximizes the value of a filled vacancy

$$J\left(z_0\right) = \max_{\{w_t(\eta^t,z^t), a_t(\eta^{t-1},z^t), s_t(\eta^t,z^t)\}_{t=0,\eta^t,z^t}^{\infty} \in \mathcal{X}} V(\mathbf{w}, \mathbf{a}, \mathbf{s}; z_0) \tag{44}$$

subject to the incentive compatibility and participation constraints described below, as well as a new set of constraints that captures limited commitment by the worker.

Incentive Constraints. The incentive compatibility condition is similar to the main text but now accounts for endogenous separation risk

$$\begin{aligned}[\mathbf{IC}] : \mathbf{a} \in \underset{\{\tilde{a}_t(\eta^{t-1},z^t)\}_{t=0,\eta^t,z^t}^{\infty}}{\operatorname{argmax}} \sum_{t=0}^{\infty} \Bigg[\int \int \beta^t \mathcal{S}_t\left(\eta^t, z^t\right) \Bigg[u\Big(w_t(\eta^t, z^t), \tilde{a}_t(\eta^{t-1}, z^t) \Big) - \Psi(s\left(\eta^t, z^t\right)) \\ + \beta s\left(\eta^t, z^t\right) \int U\left(z_{t+1}\right) \pi\left(z_{t+1} | z_t\right) dz^{t+1} \Bigg] \tilde{\pi}_t\left(\eta^t, z^t | z_0, \tilde{\mathbf{a}}\right) d\eta^t dz^t,\end{aligned} \tag{45}$$

where $\Psi(s_j)$ represents a convex utility cost to the worker of searching for a new job.

Participation Constraint. Likewise, the participation constraint must also account for separation risk and becomes

$$[\mathbf{PC}] : \sum_{t=0}^{\infty}\Bigg[\int\int \beta^t \mathcal{S}_t\left(\eta^t, z^t\right)\bigg[u\Big(w_t(\eta^t, z^t), \tilde{a}_t(\eta^{t-1}, z^t)\Big) - \Psi(s\left(\eta^t, z^t\right))$$

$$+\beta s\left(\eta^t, z^t\right)\int U\left(z_{t+1}\right)\pi\left(z_{t+1}|z_t\right)dz_{t+1}\bigg]\tilde{\pi}_t\left(\eta^t, z^t|z_0, \tilde{\mathbf{a}}\right)d\eta^t dz^t\Bigg] \geq \mathcal{E}\left(z_0\right). \quad (46)$$

Limited Commitment. Limited commitment and endogenous separations mean that after any history η^τ, z^τ the worker must rather stay in the match than separate, leading to a constraint that for each η^τ, z^τ:

$$[\mathbf{ES}] : \sum_{t=\tau}^{\infty}\mathbb{E}\Bigg[\beta^{t-\tau}\mathcal{S}_t^\tau\left(\eta^t, z^t\right)\bigg[u\Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t)\Big) - \Psi(s\left(\eta^t, z^t\right))$$

$$+\beta s\left(\eta^t, z^t\right)\mathbb{E}[U\left(z_{t+1}\right)|z_t]\bigg]\Bigg|\eta^\tau, z^\tau\Bigg] \geq U\left(z_\tau\right), \quad (47)$$

where $\mathcal{S}_t^\tau$ is the survival probability after time τ, $\mathcal{S}_t^\tau\left(\eta^t, z^t\right) \equiv \prod_{j=\tau+1}^{t}\left(1 - s_{t+\tau+1-j}\left(\eta^{t+\tau+1-j}, z^{t+\tau+1-j}\right)\right)$.

Bargaining and ex ante utility. To close the flexible incentive pay economy, we again assume ex ante utility $\mathcal{E}(z_0)$ is given by a reduced-form "bargaining schedule" $\mathcal{B}(z_0)$ as in Section A.2.

Rigid Wage Economy The rigid wage economy is identical to the rigid wage economy of the main text, including the assumption of an exogenous separation rate s.

Equilibrium Given initial unemployment u_0 and a stochastic process $\{z_t, \eta_t\}_{t=0}^{\infty}$, an equilibrium is a collection of stochastic processes $\{\theta_t, u_t\}_{t=0}^{\infty}$ and functions $J(z), U(z), \mathcal{E}(z)$, and $(\mathbf{w}, \mathbf{a}, \mathbf{s})$ such that for all firms: (i) θ_t satisfies the free entry condition so that Π_t, given in equation (42), is equal to 0 for all t; (ii) u_t satisfies the law of motion for unemployment (41); (iii) wage, effort, and separation functions $(\mathbf{w}, \mathbf{a}, \mathbf{s})$ satisfy the flexible incentive pay economy equations (44)-(47), or $w_t = \bar{w}$, $a_t = \bar{a}$ and $s_t = s$ in the rigid wage economy; (iv) the value of unemployment $U(z)$ is defined in the same way as the main text; (v) the value of employment is defined the same way as the main text for the in the rigid wage economy, or $\mathcal{E}(z) = \mathcal{B}(z)$ in the flexible incentive pay economy; and (vi) the value of a filled vacancy $J(z)$ is given by (44) in the flexible incentive pay economy or the same way as the main text for the rigid wage economy.

A.3.2 Equivalence of Rigid and Incentive Pay with Endogenous Separations

This subsection shows that, without bargaining power or fluctuations in outside options, the first-order response of market tightness is the same in the rigid wage economy and the flexible incentive pay economy with endogenous separations. For simplicity, we make the same assumptions as the main text, such as studying impulse responses in a neighborhood of the non-stochastic steady state.

Proposition 5. *Maintain Assumptions 1 and 2, and assume that the firm makes take it or leave it offers to workers, the flow value of unemployment is constant, and the optimal contract is unique. Then, the impact elasticity of market tightness to shocks to z_0 is*

$$\frac{d\ln\theta_0}{d\ln z_0} = \frac{1}{\bar{\nu}}\frac{1}{1-\Lambda} \tag{48}$$

where Λ is the steady state labor share defined as

$$\Lambda \equiv \frac{\sum_{t=0}^{\infty}\mathbb{E}\beta^t\prod_{j=1}^{t}\left(1-s_j^*\right)w_t^*}{\sum_{t=0}^{\infty}\mathbb{E}\beta^t\prod_{j=1}^{t}\left(1-s_j^*\right)f\left(z^{SS},\eta_t\right)},$$

s_j^* and w_t^* denote choices of separations and wages along the optimal contract, where the expectation $\mathbb{E}$ is evaluated along the optimal contract, and z^{SS} is the value of z_t at the aggregate steady state.

This theorem shows that the flexible incentive pay economy with endogenous separations has an equivalent response of tightness on impact to the rigid wage economy of the main text. Note that the dynamics of the rigid wage economy are still given by equation (15). Therefore, incentive wage cyclicality does not affect the impact response of tightness with endogenous separations as long as the flexible incentive pay economy and the rigid wage economy are calibrated to the same steady state labor share. In the incentive pay economy with endogenous separations, the labor share depends on the optimal choice of separation rates, as well as the factors from the model of the main text, such as wages and effort.

We stress that this result leads to equivalence for impact elasticities (Pissarides, 2009). In general, the response of unemployment to productivity shocks after impact will be different in the rigid wage and flexible incentive pay economies because endogenous separations lead to additional dynamics of unemployment after the impact of the shock.

Intuitively, in the model with efficient endogenous separations, separations are an additional choice which the firm can optimize over. However, changes in the optimal separation choice after TFP shocks have no first-order effect on profits—just as neither changes in optimally chosen effort nor wages affect profits. Likewise, the optimal contract ensures that workers do not wish to leave the match. Reoptimizations by the worker as aggregate conditions change do not affect profits. This logic is again due to the envelope theorem.

A.3.3 Proof of Proposition 5

The free entry condition in the flexible incentive pay economy is

$$\frac{\kappa}{q(\theta)} = J(z_0),$$

where $J(z_0)$ is defined in equation (44). Taking derivatives and rearranging implies

$$\begin{aligned}\frac{d\ln\theta_0}{d\ln z_0} &= \frac{1}{\nu_0}\frac{d\ln J(z_0)}{d\ln z_0} \\ &= \frac{1}{\nu_0}\frac{z_0}{J(z_0)}\frac{dJ(z_0)}{dz_0}. \end{aligned} \tag{49}$$

With Ψ convex, the optimal separation rates s_j^* will be interior. Under the assumptions of the proposition, z_0 does not enter either the incentive constraints, the participation constraint, or the limited commitment constraints directly. Therefore we have

$$\begin{aligned}\frac{dJ(z_0)}{dz_0} = \frac{\partial J(z_0)}{\partial z_0} &= \frac{\partial}{\partial z_0}\sum_{t=0}^{\infty}\mathbb{E}\beta^t\prod_{j=1}^{t}\left(1-s_j^*\right)\left(f(z_t,\eta_t)-w_t\right) \\ &= \sum_{t=0}^{\infty}\mathbb{E}\beta^t\prod_{j=1}^{t}\left(1-s_j^*\right)\left(f_z(z_t,\eta_t)\right), \end{aligned} \tag{50}$$

where the first equality invokes the envelope theorem, using the same argument as the main Appendix and also using our assumption of a unique optimal contract in order to dispense with a sup operator; the second equality rewrites the definition of profits from equation (43) using the notation from the theorem and exploits that terms involving the participation, incentive, or limited commitment constraints vanish because z_0 does not enter them directly; and the final equality uses that z_t is a random walk.

Substituting in equations (49) and (50) implies

$$\begin{aligned}
\frac{d\ln\theta_0}{d\ln z_0} &= \frac{1}{\nu_0}\frac{z_0\sum_{t=0}^{\infty}\mathbb{E}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f_z(z_t,\eta_t)\right)}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f\left(z_t,\eta_t\right)-w_t^*\right)\right]} \\
&= \frac{1}{\nu_0}\frac{z^{SS}\sum_{t=0}^{\infty}\mathbb{E}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f_z(z^{SS},\eta_t)\right)}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f\left(z^{SS},\eta_t\right)-w_t^*\right)\right]} \\
&= \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\mathbb{E}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f(z^{SS},\eta_t)\right)}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f\left(z^{SS},\eta_t\right)-w_t^*\right)\right]} \\
&= \frac{1}{\nu_0}\frac{1}{1-\frac{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)w_t^*\right]}{\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f(z^{SS},\eta_t)\right)\right]}},
\end{aligned}$$

where we use the assumption of an aggregate steady state at $z = z^{SS}$. □

B Numerical Appendix

This appendix studies a quantitative version of our flexible incentive pay economy. Appendix Section B.1 calibrates the model. Appendix Section B.2 uses the model for three purposes. First, we try to disentangle the share of total wage cyclicality that arises for non-incentive versus incentive reasons. Second, we show that thanks in part to incentive pay, our model can quantitatively match unemployment and productivity dynamics. Third, we numerically establish that the equivalence between rigid wage and incentive pay economies holds globally, for large productivity shocks. Appendix Section B.3 includes additional details on our solution algorithm.

B.1 Numerical Analysis: Calibration

Parameterizing the model. We parameterize the production function, utility function, ex ante utility, and information structure of the flexible incentive pay model of Section 2 following Edmans et al. (2012), allowing for cyclical ex ante utility as in section A.2.

Production function. The firm's production function is $y = z(a + \eta)$. Idiosyncratic profit shocks η are assumed to be i.i.d. over time and across individuals and normally distributed with zero mean and standard deviation σ_η. σ_η determines the extent to which firms can infer workers' effort, which is key for incentive pay.[19]

Preferences. We assume that workers' utility function is given by $u(c, a) \equiv \ln c - \frac{a^{1+1/\epsilon}}{1+1/\epsilon}$. ϵ governs the Frisch elasticity of effort, which determines how costly effort is to workers.

[19] In reality, idiosyncratic shocks are likely persistent. Unfortunately, solving dynamic incentive contract models with persistent shocks is not currently computationally feasible.

Information structure. We assume that η is not observed to the firm; it therefore plays the role of $\tilde{\eta}$ in the static limit studied in section 3.1 as firms can infer $a + \eta$ each period when they observe y. We make the "effort after noise" assumption as in Edmans et al. (2012): workers observe the idiosyncratic profit shock η before making an effort choice. Thus, there is an incentive compatibility constraint for each value of η. Following Edmans et al. (2012), we assume that a unique level of effort $a(z^t)$ is implemented regardless of the idiosyncratic shock η.[20] However, effort varies with the history of aggregate productivity z^t.

Ex ante utility. We assume that firms make take-it-or-leave-it offers to workers who face procyclical unemployment benefits. Workers' flow unemployment benefits take the form $b(z) = \gamma z^{\chi}$. Here, γ specifies the level of unemployment benefits when $z = 1$, while χ determines the elasticity of unemployment benefits to aggregate productivity. This specification is a log-linear approximation of any differentiable $\mathcal{B}(z)$ function, including models in which workers and firms bargain over ex ante utility at the start of the contract. However, this specification is numerically tractable in that it abstracts from complications of bargaining and ensures that unemployed workers' value is given by the present discounted value of expected unemployment benefits. The parameter χ represents all the reasons why the utility promised to workers at the beginning of an employment relationship may be cyclical—such as fluctuations in either the worker's outside option (changes in the value of unemployment) or her inside option (bargained utility)—and determines NWC.

We now characterize the behavior of wages under the optimal contract following Edmans et al. (2012), which will be useful for motivating our calibration strategy.

Proposition 6. *The earnings schedule in the optimal contract satisfies the following difference equation (given initial productivity z_0):*

$$\ln(w_t(\eta^t, z^t)) = \ln(w_{t-1}(\eta^{t-1}, z^{t-1})) + \psi h'(a_t)\eta_t - \frac{1}{2}(\psi h'(a_t)\sigma_\eta)^2, \tag{51}$$

where $\psi = 1 - \beta(1 - s)$ and $w_{-1}(z_0)$, which initializes the difference equation, is defined in the proof of the proposition.

The proof closely follows those of Edmans et al. (2012) and Doligalski et al. (2023).[21] Equation (51) characterizes wage growth. The pass-through of idiosyncratic shocks to wages, $\psi h'(a_t)\eta_t$, corresponds to incentives. If the marginal disutility of effort h' is high, there must

[20] This assumption differs from the setup of the model of Section 2 and is for computational tractability. As Edmans et al. (2012) discuss, without this assumption, a closed form for the optimal contract is unavailable, which prohibits simulating the model.

[21] We make three advances relative to Edmans et al. (2012): we introduce aggregate risk z_t, allow for pro-cyclical ex ante worker utility, and develop a global solution algorithm to efficiently simulate the model with labor market search.

be a high pass-through from η to wages to induce workers to supply the optimal effort level. To satisfy dynamic incentives, the pass-through of idiosyncratic productivity shocks to wages is scaled down by a quantity ψ that reflects discounting. Exponentiating equation (51), one observes that wages are a random walk: the expectation of wages in period $t+h$ is equal to the level of wages in period t. The random walk property is a consequence of the inverse Euler equation (Rogerson, 1985). Thus, rescaled expected wages at the start of the job, $w_{-1}(z_0)/\psi$, equal the expected present discounted value (EPDV) of wage payments.

Effort is determined by the first-order condition of the firm's maximization problem:

$$J(z_0) = \max_{\{a_{it}, w_{-1}(z_0)\}} \mathbb{E}_0 \left[\sum_t z_t (a_{it} + \eta_{it}) \right] - \frac{w_{-1}(z_0)}{\psi},$$

subject to the participation constraint, where the worker's expected utility under the contract can be solved for using equation (51). The optimal effort schedule, given an initial z_0 and current z_t, thus satisfies

$$a\left(z_t; z_0\right) = \left[\frac{z a\left(z_t; z_0\right)}{w_{-1}(z_0)} - \frac{\psi}{\epsilon} \left(h'\left(a\left(z; z_0\right)\right) \sigma_\eta\right)^2 \right]^{\frac{\epsilon}{1+\epsilon}}.$$

Free entry into vacancy posting requires that EPDV of profits, $Y(z_0) - w_{-1}(z_0)/\psi$, equals the expected cost of filling a vacancy $\kappa/q(\theta)$, where $Y(z_0)$ is the EPDV of output. This implies that $w_{-1}(z_0) = \psi(Y(z_0) - \kappa/q(\theta(z_0)))$.

Calibration: Separating bargaining and outside options from incentives. Our goal is to infer the role of bargaining and outside options versus that of incentives in determining wage cyclicality. We disentangle these forces with two sets of moments: the cyclicality of the wage for new hires, which informs non-incentive wages, and the pass-through of idiosyncratic firm output shocks into wages as well as the variance of workers' wage growth, both of which inform incentives. That is, wage fluctuations after the start of the match inform incentives, while wage fluctuations at the start of the contract inform NWC.

We calibrate the parameters of the labor search block largely following the standard practice of Shimer (2005) and Petrosky-Nadeau and Zhang (2017).[22] Productivity is assumed to follow an AR(1) process in logs, with autocorrelation parameter ρ_z, innovation $\zeta_t \sim \mathcal{N}(0, \sigma_z^2)$, and mean μ_z. We discretize the AR(1) productivity process for $\ln z_t$ onto a finite grid: $z \in \mathcal{Z} = [\underline{z}, ..., \overline{z}]$ following Rouwenhorst (1995). We set the number of gridpoints to 13. We normalize μ_z such that $\mathbb{E}[z_t] = 1$. To account for the effects of effort fluctuations on

[22] These parameters are the discount rate, the vacancy creation cost, the matching function, and the separation rate. We discuss the details in Appendix Section B.3.

productivity, we calibrate our monthly process for z such that the log of the quarterly average of z_t matches the autocorrelation and standard deviation of the quarterly log TFP series described in Fernald (2014), which accounts for variable capacity utilization in labor. We view the TFP series net of variable capacity utilization as a reasonable proxy for exogenous productivity, as labor utilization is a concept highly related to effort.[23] This procedure implies a monthly autocorrelation $\rho_z = 0.965$ and standard deviation of shocks $\sigma_z = 0.0056$.[24]

This leaves four parameters to internally calibrate: the variance of noise σ_η, the level and cyclicality of ex ante utility χ and γ, and the effort elasticity ϵ. We target the variance of incumbent wage growth, the pass-through of firm shocks into wages, the cyclicality of new hire wages, and the average unemployment rate. While we estimate all parameters jointly, these moments have intuitive mappings to particular parameters, which we explore below.

First, the variance of wage growth naturally informs the variance of idiosyncratic profit shocks σ_η. To see this, note that rearranging equation (51) shows that the monthly wage growth of job-stayers is given by $\Delta \ln w_t = \psi h'(a_t)\eta_t - 1/2\left(\psi h'(a_t)\sigma_\eta\right)^2$. At an aggregate non-stochastic steady state, $a_t = a^{SS}$, for example, the cross-sectional variance of wage growth is given by $Var(\Delta \ln w) = \psi^2 h'(a^{SS})^2\sigma_\eta^2$, which is closely tied to the value of σ_η. The firm provides *intertemporal incentives* by exposing the worker to wage-growth risk as in Sannikov (2008). We target a standard deviation of year-over-year wage growth of job-stayers of 0.064 as measured by Grigsby et al. (2021), where we calculate year-over-year wage growth in the model with stochastic z_t by iterating on equation (51) for job-stayers.[25]

Second, the pass-through of firm-specific shocks to wages is informative of whether incentives are high powered within the contract, as in classic theories of moral hazard. In particular, this pass-through helps us identify the parameter governing the disutility of effort ϵ. In our model, the expected pass-through from idiosyncratic output shocks to the wages of job-stayers is given by $\mathbb{E}\left[\partial \ln w/\partial \ln y\right] = \mathbb{E}\left[\psi h'(a)(a+\eta)\right]$, which is directly affected by $h'(a)$. The firm provides *intratemporal incentives* with the pass-through of output to wages. Intuitively, if $h'(a)$ is high, then workers would prefer not to supply more effort, so the firm must make wages highly dependent on output to incentivize effort.

Pass-through of firm-level output shocks to wages is evidence that firm-specific labor supply curves are upward-sloping. This is commonly interpreted as evidence that firms have some degree of monopsony in the labor market (Manning, 2003; Card et al., 2018; Berger et

[23]Basu and Kimball (1997) find that variable capacity utilization explains approximately 40–60% of fluctuations in unadjusted TFP and that capacity utilization is procyclical.

[24]We HP-filter the TFP data and model-simulated series with a smoothing parameter of $\lambda = 10^5$, following Shimer (2005), which removes a very low-frequency trend.

[25]Hours are observable and thus contractible. We therefore consider earnings per hour— including base pay, bonuses, and overtime—to be the correct empirical counterpart of w_t.

al., 2022). In the textbook monopsony model, a firm chooses a labor input internalizing the labor supply curve it faces. The reason that firm-level shocks passthrough to wages is that the firm must trade up its labor supply curve in order to increase its desired labor input; since the curve is upward-sloping, the firm must raise wages to increase labor input.

These economics are nearly identical to those in the incentive pay model. Here, the firm-specific labor supply curve is given by the incentive compatibility and participation constraints. The passthrough informs the power of incentives precisely because it reveals the slope of the firm-specific labor supply curve; that is, how much must the firm raise wages in order to receive one more unit of the labor input: effort. Therefore, the reason why firms pass through output shocks to incumbent workers' wages is the same in both the monopsony and incentive models. If "labor input" in the monopsony model occurs on the intensive margin of output per worker — such as hours or effort movements — then there is little distinction between the monopsony and incentive models. Indeed, passthrough reflects exactly the extent to which wage movements are offset by movements in output per worker in both models. Thus the passthrough moment remains an appropriate choice for our purposes.

However, if the labor input in the monopsony model occurs on the extensive margin by hiring workers, then any passthrough to incumbent workers' wages would constitute a rent those workers receive. Some of the passthrough in the data may reflect these kinds of considerations; we therefore target a low estimate of the passthrough from the literature.

A large literature seeks to estimate the pass-through to job-stayers' wages of firm-specific profitability shocks; Card et al. (2018) provides a comprehensive survey. We target an average pass-through of firm-level output shocks to wages of 0.039, the value estimated in Martins (2009), which is on the low end of the range reported by Card et al. (2018), which spans from 0.020 to 0.290. Our targeting of a low pass-through value is likely to be a conservative choice, as low pass-through suggests that incentives are not high powered and therefore are a relatively unimportant determinant of wage variation.

Third, we identify γ, which pins down the level of unemployment benefits from the average level of unemployment. Average unemployment is determined by workers' job finding rates, which in turn are determined by expected profits per worker. γ directly influences expected profits because it governs workers' value of unemployment and shifts the level of the required wage payments to workers. We target an average unemployment rate of 6%, consistent with average U.S. unemployment between 1951 and 2019.

Fourth, we target the cyclicality of new hire wages to inform the cyclicality of nonemployment benefits χ. Conditional on the parameters governing incentives, the cyclicality of new hire wages is highly informative of χ. Intuitively, if the worker's outside option is highly procyclical, so too is her promised utility, and thus, so too will be the present value of her wages. Since wages are a random walk in the optimal contract by equation (51), the cyclicality of new hire wages strongly informs the cyclicality of the present value of wages. We target a semielasticity of new hire wages to unemployment of -1, which is at the high end of the range found by Grigsby et al. (2021) and Hazell and Taska (2022), and explore the robustness of our findings to this choice.

Our model links ex post wage pass-through to incentives and not to Nash bargaining. In the face of this particular concern, we target a conservative value of pass-through. Moreover, there is empirical evidence that pass-through is procyclical (Chan et al., 2023), which is consistent with our model and inconsistent with pass-through representing Nash bargaining.

The intuition for our estimation is as follows. The cyclicality of new hire wages is highly informative about the cyclicality of the present discounted value of wage payments over the life of a match, because log wages are a random walk on the optimal contract. It then remains to determine the share of wage cyclicality that is matched with movements in effort. Two parameters are key for this. First, σ_η governs the degree to which effort can be inferred from realized output: if $\sigma_\eta = 0$, then output directly corresponds to an effort movement, while if $\sigma_\eta \to \infty$, effort has no impact on idiosyncratic output. This parameter is thus crucial for the degree to which effort must be incentivized rather than enforced. As discussed above, the variance of wage growth on the job is highly informative for this parameter. Second, ε governs the cost of incentive provision on the margin. If ε is high, then workers strongly dislike providing additional effort on the margin. As a result, the firm must increase expected wages a great deal in order to incentivize worker effort. As discussed above, the passthrough of idiosyncratic shocks to wages is informative about this parameter. Therefore, three moments—new hire wage cyclicality, on-the-job variance of wage growth, and the passthrough of idiosyncratic shocks to wages—principally determine the key parameters for the share of wage cyclicality due to incentives (σ_η, ε and χ).

Calibration results. Table B1 summarizes our calibration, while Table B2 examines the implications for various moments. We estimate that the elasticity of the disutility of effort ϵ is 3. Note that standard estimates of micro labor supply elasticities, such as those computed by Chetty (2012), consider how hours vary with wages. Since hours are observable and contractible by the firm, the lower elasticities of hours need not have any relationship with the elasticity of unobservable effort. Intuitively, one might expect the elasticity of effort to be larger than that of hours: while many jobs have a fixed number of hours over which the worker has little control, workers may be able to adjust unobserved effort more elastically.

We find the level of unemployment benefits γ to be 0.45. This value is close to the value

Table B1: Calibrated parameter values

Parameter	Description	Value	Source/Target
Externally Calibrated			
β	Discount Rate	0.997	Petrosky-Nadeau and Zhang (2017)
κ	Vacancy Creation Cost	0.450	Petrosky-Nadeau and Zhang (2017)
s	Separation Rate	0.031	CPS E, U Stocks (following Shimer (2005))
ρ	Autocorrelation: Agg. Productivity	0.965	Autocorrelation: Fernald (2014) TFP
σ_z	Cond. S.D. of Agg. Productivity	0.006	Uncond. S.D.: Fernald (2014) TFP
Internally Calibrated			
γ	Level: Unemployment Benefits	0.453	Average Unemployment Rate
ϵ	Elasticity: Disutility of Effort	2.999	Pass-Through: Profits to Wages
σ_η	S.D.: Idiosyncratic Profit η	0.538	S.D.: Job-Stayer Log Wage Growth
χ	Cyclicality: Promised Utility to Worker	0.462	New Hire Wage Cyclicality

chosen by Shimer (2005) to match the replacement rate of unemployment benefits (0.4).[26]

We estimate the standard deviation of idiosyncratic profit shocks to be $\sigma_\eta = 0.54$, similar to estimates in other labor search calibrations with idiosyncratic shocks (e.g., Schaal, 2017). This, coupled with a sizable elasticity of effort, suggests that incentive provision is a relatively important consideration for the firm. We estimate the cyclicality of flow unemployment benefits χ to be 0.46, implying moderately procyclical promised utility to the worker.[27]

Table B2 compares key moments in both the data (Column (1)) and calibrated model (Column (2)). The top panel reports the moments that we target in the estimation. The model is able to fit the targeted moments very well. Most notably, we match the cyclicality of new hire wages almost exactly and, if anything, underestimate the pass-through of firm shocks to wages, suggesting that our estimate of the importance of incentives for wage cyclicality is likely a lower bound on its true importance.

The bottom panel of the table shows that the model generates approximately half of the unconditional volatility of aggregate unemployment observed in the data. One should expect a quantitatively successful calibration to only match half of the unemployment volatility. The reason is that, as Hall and Milgrom (2008) point out, in the data not all unemployment fluctuations correlate with output per worker fluctuations. In both our model and the standard Diamond-Mortensen-Pissarides model, all unemployment fluctuations correlate with output per worker fluctuations. Other shocks must account for the remaining unemployment fluctuations. As such, Hall and Milgrom (2008) argues that one should evaluate whether the model can explain the portion of unemployment volatility that, in the data, correlates with

[26]Note, however, that unemployed workers do not need to supply effort in this model, which increases the effective flow unemployment value.

[27]Chodorow-Reich and Karabarbounis (2016) estimate $\chi \approx 0.8$; however, the value of unemployment in our model is different from theirs because workers supply effort and do not have access to financial assets.

Table B2: Model fit to data moments

Moment	Description	Data (1)	Model (2)
Targeted			
$d\mathbb{E}[\ln w_0]/du$	Cyclicality of new hire wages	-1.000	-1.001
$\mathbb{E}[\partial \ln w_{it}/\partial \ln y_{it}]$	Within-job pass-through of idiosyncratic shock	0.039	0.037
$\text{std}(\Delta \ln w_{it})$	std(ln wage growth for job-stayers)	0.064	0.064
$\bar{u}_t$	Mean unemployment	0.060	0.060
Untargeted			
std $(\ln u_t)$	Volatility of unemployment (quarterly)	0.203	0.101
Incentive share	Share of wage cyclicality due to incentives	–	0.469

output per worker. Hall and Milgrom (2008) calculate that the volatility of the component of unemployment that correlates with output per worker is 0.68 percentage points. Our model predicts that the volatility of unemployment is $0.101 \times 6\% = 0.61pp$, though our sample is somewhat different from Hall and Milgrom (2008). Therefore, even though our main focus is the impulse response of unemployment, our calibrated model does match unconditional unemployment fluctuations reasonably well. Matching the micro moments of wage adjustment therefore generates significant unemployment volatility, the reasons for which we will discuss shortly.

B.2 Numerical Results

Incentive wage cyclicality. Now, we discuss our key numerical result: the model suggests that a significant share of wage cyclicality is due to incentives. As a result, unemployment responds strongly to business cycle shocks despite wages being relatively procyclical.

The model calibration reveals in the final row of Table B2 that approximately 47% of the total wage cyclicality is due to incentives. This may seem large. Non–base compensation, which may be associated with incentives, is relatively small for most workers. However, what matters for wage cyclicality is whether the *marginal* dollar of wages paid is due to incentives or bargaining and outside options. If, for instance, 2% of compensation is incentive pay in the steady state but only incentive pay is cut in response to output shocks, then the share of wage cyclicality attributable to incentives is 100%. Further, base wages may embed some incentive components if workers can be promoted after good performance; indeed, promotions appear to be quite cyclical in the data (Méndez and Sepúlveda, 2012; Wilson, 1996; Addison et al., 2014). These kinds of incentives can explain why incentive wages may

Table B3: Model moments: Alternative calibrations

Moment	Model: Source of wage flexibility (1) Incentives + Bargaining	(2) Incentives	(3) Bargaining	(4) Bargaining: $\partial\mathbb{E}[\ln w_0]/\partial u = -0.53$
$d\mathbb{E}[\ln w_0]/du$	-1.00	-0.59	-1.00	-0.53
$d\ln\theta_0/d\ln z_0$	13.59	18.10	10.27	13.31
std($\ln u_t$)	0.10	0.15	0.07	0.10
$\mathcal{W}_0/\mathcal{Y}_0$	0.96	0.96	0.96	0.96
$d\ln\mathcal{W}_0/d\ln z_0$	0.44	0.38	0.31	0.23
$d\ln\mathcal{Y}_0/d\ln z_0$	0.70	0.87	0.50	0.50
NWC share	0.53	0.00	1.00	1.00

Notes: New hire wage cyclicality is targeted, while the second set of moments is untargeted. Column (1) is our baseline model. Column (2) sets $\chi = 0$ and does not target the cyclicality of new hire wages. Columns (3) and (4) fix effort $a = 1$, set wages to be constant within the contract, and do not target the standard deviation of wage growth or the pass-through. Column (4) targets a cyclicality of new hire wages of -0.46. The standard deviation of log unemployment is computed at the quarterly frequency. For the untargeted moments, x_0 denotes the value of variable x, evaluated at $\ln z = \mu_z$. $\mathcal{W}$ and $\mathcal{Y}$ refer to the expected present value of wage payments and output, respectively. "NWC share" is the share of wage cyclicality not accounted for by incentives, also evaluated at $\ln z = \mu_z$.

be quite cyclical even if measured bonuses are relatively acyclical.[28]

Because NWC is relatively small, the impulse response of unemployment to business cycle shocks is relatively large. Table 3 reports a number of additional features of our model calibrated in a variety of ways. Column (1) reproduces the baseline calibration as in Table B2. The impulse response of market tightness to business cycle shocks is in the second row. Market tightness responds greatly to exogenous productivity shocks: the elasticity of market tightness to aggregate productivity is 13.6.

This occurs despite total wages being quite procyclical. The conditional elasticity of the present value of expected wage payments with respect to productivity is 0.44. However, as we have discussed in previous sections, the stabilizing effect on unemployment of procyclical wages is offset by the amplifying effect of effort and incentives. Because of incentives, the impulse response of the present value of output, $\mathcal{Y}_0$, to TFP shocks is a relatively large value of 0.70. As a result, profit fluctuations—and thus market tightness and employment fluctuations—are large despite the procyclicality of wages.[29] The model implies a labor share

[28]There is debate about how cyclical bonuses are in the data. Grigsby et al. (2021) find very limited bonuses cyclicality in recent administrative payroll data, while Swanson (2007) and Shin and Solon (2007) both find that non-based pay is quite cyclical in longer-run survey data. We choose not to target measured cyclicality of bonuses both due to this empirical debate, and because bonuses need not reflect incentives; see, for instance, Oyer (2004) for a theory in which bonuses do not have strong incentive effects.

[29]We caution that this statistic does not have an easy-to-measure empirical counterpart, given the challenge of measuring either TFP shocks or the present value of output well.

(defined as $\mathcal{W}_0/\mathcal{Y}_0$) of 0.96, in line with, for instance, Hall (2005).[30]

To emphasize the role of incentive wage cyclicality, we consider versions of our model that load all wage cyclicality in the data onto either incentives or bargaining and outside options. We present the calibration with only incentives in Column (2), which leads to a large impulse response of tightness in row 2.[31] Nevertheless, the incentives-only model still generates large wage cyclicality in row 1. This is a manifestation of our analytical results in a globally solved model. Column (3) presents a version of the model without incentives and with only bargaining, where the impulse response of tightness is relatively small, reflecting the dampening effect of NWC.[32]

The distinction between bargaining and incentives also matters for unemployment volatility. If all wage cyclicality were due to bargaining, then the standard deviation of log unemployment would be 0.07. Therefore if all wage cyclicality is due to bargaining, unemployment volatility is 30% lower than in the baseline calibration, which we have argued is reasonably consistent with the data.

Overall, in our calibration, incentive pay appears to be important. With this feature, one can simultaneously match a relatively high estimate of wage cyclicality, alongside volatile unemployment. We stress that other mechanisms can also match both of these facts, such as a small "fundamental surplus" (Ljungqvist and Sargent, 2017).

Calibrating simpler models. We argue that the simple version of our model in which all wage cyclicality is due to bargaining should target a new hire wage cyclicality given only by NWC—that is, a calibration in which wages are less procyclical than in the data. To illustrate the point numerically, we recalibrate the bargaining-only version of the model targeting a new hire wage cyclicality of -0.53, which is what we previously inferred to be non-incentive wage cyclicality.[33] Column (4) of Table B3 presents the results of this exercise.

The numerical results show that to produce the correct impulse response of market tightness in the simple model with only bargaining, calibrating to target NWC is crucial. When calibrated to NWC, the bargaining-only model features an elasticity of market tightness to exogenous shocks that is nearly identical (13.3) to that in the full model (13.6). Furthermore, both models generate an unconditional standard deviation of log unemployment rates

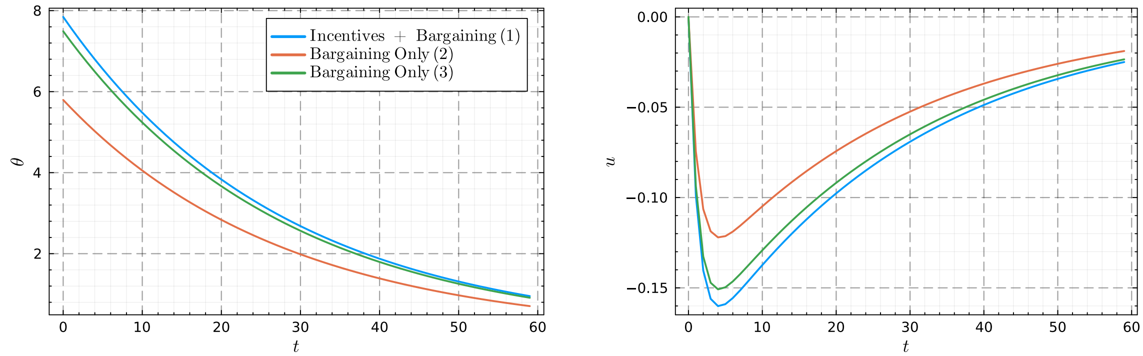
[30]Since our model does not have capital, the labor share corresponds to the labor share of payroll and rents from search frictions in the labor market, excluding capital (Pissarides, 2000).

[31]This calibration assumes that the cyclicality of ex ante utility is zero and do not target wage cyclicality.

[32]This calibration turns off incentives by setting the variance of the idiosyncratic profitability shocks to $\sigma_\eta = 0$, exogenously fixing effort $a = 1$, setting $\epsilon = 1$, and setting wages to be fixed within a contract. We attribute all wage cyclicality in the data to the cyclicality of promised utility governed by χ.

[33]We normalize $\epsilon = 1$ for this exercise and solve for fixed wages within the contract. We also drop the standard deviation of log wage growth and the average pass-through of firm shocks as targeted moments.

Figure B3: Impulse response to shock to z_0 with bargaining-only and incentive pay models



Panel A: Tightness θ_t Panel B: Unemployment u_t

Notes: The figure shows impulse responses for five years after a one-standard-deviation shock to z_0. In Panel A, θ_t is shown in percentage deviations from the steady state. In Panel B, u_t is shown as deviations away from the steady state in percentage points. All models are calibrated as in B3.

of 0.10. The similar dynamics arise because the two models imply similar ex ante utility cyclicality even though overall wage flexibility is different: the simple bargaining-only model of column (4) estimates an elasticity of unemployment benefits $\chi = 0.46$, nearly identical to that found under the full model.

Figure B3 plots the impulse of market tightness (Panel A) and unemployment (Panel B) in response to a one-standard-deviation increase in aggregate productivity z, which decays according to an AR(1).[34] The blue line is the response in the full model with both incentives and bargaining. The red line is the response in the bargaining-only model calibrated to the full wage cyclicality in the data. The green line is the response in the bargaining-only model calibrated to our estimate of NWC in the data. The response of both market tightness and unemployment is approximately 25% less pronounced in the bargaining-only model than in the full model with incentives and bargaining. However, the impulse responses of both tightness and unemployment are nearly identical in the full model and the bargaining-only model calibrated to relatively rigid wages.

Robustness. The key numerical result of this section is that a significant share of wage cyclicality is due to incentives, leading to volatile unemployment dynamics despite wages being relatively procyclical.

[34]Further details on the construction of these impulse responses are described in Section B.3.5.

Table B4: Alternative calibration strategies: Internally calibrated parameters

Parameter	$\partial\mathbb{E}[\ln w_0]/du$ target				Internal Calibration: ALP	
	-0.5	-0.75	-1.25	-1.5	Incentives + Bargaining	Bargaining
σ_η	0.536	0.537	0.538	0.538	0.533	0.000*
χ	0.180	0.344	0.547	0.610	0.480	0.609
γ	0.475	0.452	0.453	0.453	0.585	0.584
ϵ	2.325	3.000	2.999	3.000	0.966	1.000*
ρ_z	0.965*	0.965*	0.965*	0.965*	0.990	0.969
σ_z	0.006*	0.006*	0.006*	0.006*	0.003	0.006

Notes: Table reports estimated parameters for our alternative calibration strategies. The first four columns change the target of new hire wage cyclicality for our full model with both incentives and bargaining. The final two columns internally calibrate the exogenous productivity process to match moments of measured labor productivity under our full model and model with only bargaining. Asterisks indicate fixed parameters.

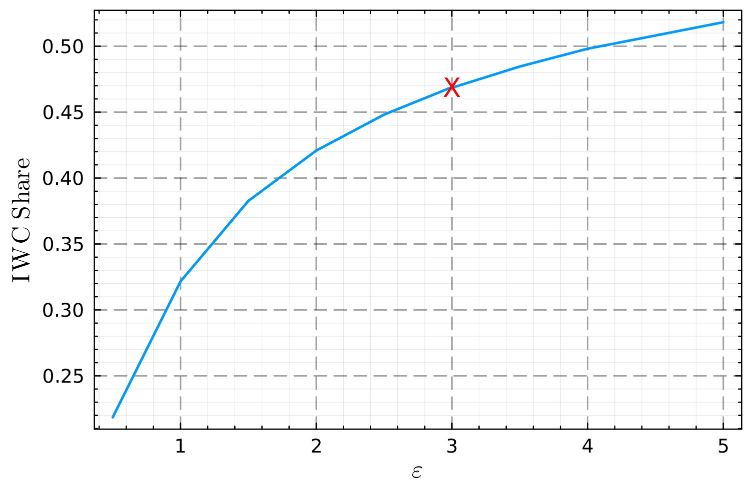
Table B5: Varying cyclicality of new hire wages: Simulated model moments

	Model: $\partial\mathbb{E}[\ln w_0]/du$ target			
Moment	-0.50	-0.75	-1.25	-1.50
$d\mathbb{E}[\ln w_0]/du$	-0.50	-0.75	-1.25	-1.50
std($\ln u_t$)	0.16	0.13	0.09	0.07
$d\ln\theta_0/d\ln z_0$	18.18	15.79	11.83	10.44
Incentive Wage Cyclicality share	0.76	0.59	0.39	0.33
Incentive Wage Cyclicality	-0.38	-0.44	-0.48	-0.49

Notes: New hire wage cyclicality is targeted, while the second set of moments are untargeted. std($\ln u_t$) is the unconditional standard deviation of the log of the quarterly average of the monthly unemployment rate, HP-filtered with smoothing parameter $\lambda = 10^5$. x_0 denotes the value of variable x, evaluated at $\ln z = \mu_z$. Incentive Wage Cyclicality share is the share of wage cyclicality that is due to incentives, also evaluated at $\ln z = \mu_z$. Incentive Wage Cyclicality is computed as $\mathbb{E}[\ln w_0]/du$ × Incentive Wage Cyclicality share.

Tables B4 and B5 report the estimated parameters and model-implied moments, respectively, when we target different values of wage cyclicality ranging from -0.5 to -1.5. Each column corresponds to a recalibration of the model. We find that the share of wage cyclicality attributable to incentives declines as we increase the target estimate of cyclicality of new hire wages. However, Incentive Wage Cyclicality is always large and relatively stable between -0.38 and -0.49.

Figure B5: Incentive wage cyclicality share for different calibrations of ϵ



Notes: Figure reports the estimated share of wage cyclicality due to incentives at $\ln z_0 = \mu_z$ as we vary the disutility of effort ϵ. To produce this figure, we first impose a value of ϵ and then recalibrate our model to match all four of our baseline calibration targets.

To account for uncertainty in our pass-through target, Figure B5 reports the estimated value of the incentive wage cyclicality share for various imposed values of ϵ, allowing all other parameters to be recalibrated. The X on the plot reports our baseline estimate for ϵ. The estimated share of wage cyclicality due to incentives is increasing in ϵ, rising to 52% for $\epsilon = 5$ and falling to 22% for $\epsilon = 0.5$.

One concern with our calibration is as follows. Our empirical measure of productivity, from Fernald (2014), may not be meaningful in this environment. The measure of productivity accounts for variable capacity utilization. However in our model, one requires a measure of productivity that accounts for worker effort, which is different from capacity utilization. Therefore, we adopt a second calibration strategy. We internally calibrate the exogenous productivity process in our incentive pay model to match moments of average labor productivity (ALP) in the data.[35] Tables B4 and B6 report the estimated parameters and model-implied moments, respectively. Calibrated thus, the model infers that incentives account for about 35% of overall wage cyclicality and a large impulse response of market tightness to exogenous productivity shocks. Moreover the model predicts log unemployment volatility of 0.09, compared with log unemployment volatility in the data of 0.2. As we have discussed, an appropriate calibration should explain roughly half of the unemployment volatility in the data.[36]

[35]Here, ALP is the seasonally adjusted, quarterly average output per hour for all workers in the nonfarm business sector, as reported by the BLS.

[36]In Appendix Tables B4 and B6, we also recalibrate the bargaining-only model to target average labor productivity. The bargaining-only model continues to have a significantly smaller impulse response of

Table B6: Internally calibrating labor productivity process: simulated model moments

		Model: Source of wage flexibility	
Moment	Data	Incentives + Bargaining	Bargaining
ρ_p	0.89	0.87	0.88
σ_p	0.02	0.02	0.02
$\mathrm{std}(\ln u_t)$	0.20	0.09	0.08
${d \ln \theta_0}/{d \ln z_0}$	-	18.94	10.78
${\mathcal{W}_0}/{\mathcal{Y}_0}$	-	0.96	0.96
${d \ln \mathcal{W}_0}/{d \ln z_0}$	-	0.58	0.33
${d \ln \mathcal{Y}_0}/{d \ln z_0}$	-	0.99	0.54
Incentive share	-	0.35	0.00

Notes: New hire wage cyclicality is targeted, while the second set of moments are untargeted. $\mathrm{std}(\ln u_t)$ is the unconditional standard deviation of the log of the quarterly average of the monthly unemployment rate, HP-filtered with smoothing parameter $\lambda = 10^5$. ρ_p and σ_p are the autocorrelation and unconditional variance of the log of quarterly average labor productivity, HP-filtered with smoothing parameter $\lambda = 10^5$. x_0 denotes the value of variable x, evaluated at $\ln z = \mu_z$.

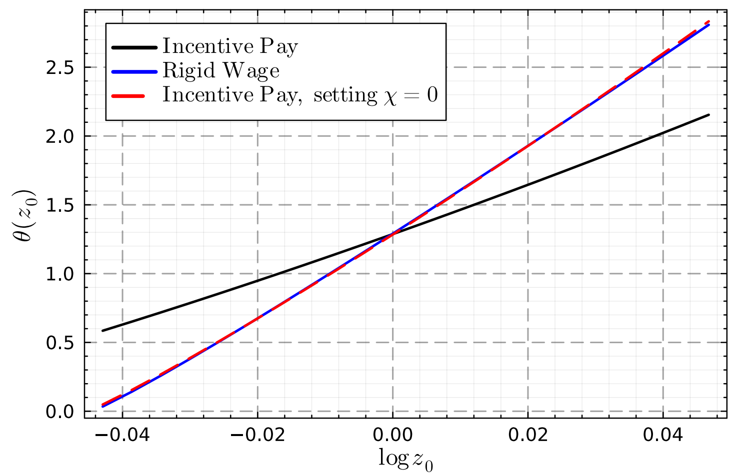
Taking stock, we find that a relatively large share of wage cyclicality in the data is attributable to incentives despite our conservative calibration. Therefore, our model generates a large impulse response of unemployment despite the cyclicality of wages.

Global Equivalence. Our theoretical result is first-order in nature: it shows that the response of market tightness to exogenous productivity shocks is the locally same in the flexible incentive pay model with acyclical promised utility as in an appropriately-calibrated model with rigid wages and effort. Figure B6 studies the extent to which this holds globally in the quantitative model. It plots movements in market tightness θ as aggregate productivity z_0 varies for up to 5% aggregate TFP shocks.

The figure contains three lines. The red line reports the relationship between θ and z_0 for the flexible incentive pay model, calibrated exactly as in Table B1 only with $\chi = 0$ so that there is no non-incentive wage cyclicality. The blue line plots this relationship in the rigid wage model, calibrated such that the present value of wages and output are the same in the rigid wage and incentive pay models at $z_0 = 1$. This calibration ensures that the blue and red lines intersect at $\log z_0 = 0$ by construction, since the labor share at that point is given by 0.96 in both economies. The content of our theorem is that the blue and red lines lie on top of each other, not just at $\log z_0 = 0$, but also for movements in z_0 of at least 5%; that is, the flexible incentive pay and rigid wage models generate the same movements in θ (and

tightness than does the full model and requires exogenous productivity shocks to be approximately twice as volatile as in the full model to match output fluctuations.

Figure B6: Tightness Fluctuations in Quantitative Model



Notes: Figure plots market tightness θ as a function of aggregate productivity z_0 in the quantitative model model. The black line considers the calibrated flexible incentive pay economy with non-zero non-incentive wage cyclicality. Calibration follows Table B1. The red line considers the flexible incentive pay economy without non-incentive reasons for wage cyclicality, generated by setting $\chi = 0$. The blue line plots considers the rigid wage economy, calibrated to have the same present value of wages and output as the flexible incentive pay economy for $z_0 = 1$.

thus unemployment) even for large shocks to aggregate productivity. Theorem 1 numerically appears to not only hold locally, but for larger shocks as well.

As a point of comparison, the black line on Figure B6 reports the response of θ to z_0 for the full model with non-zero non-incentive wage cyclicality (that is, $\chi = 0.46$ as in Table B1). The plot makes clear that non-incentive wage cyclicality dampens the relationship between market tightness θ and exogenous productivity shocks $\log z_0$, as suggested by Proposition 4.

B.3 Numerical Details

We calibrate the model such that t represents a month. Specifically, we set the discount rate β to $0.99^{1/3}$, the vacancy creation cost to 0.45 and employ a matching function given by $m(u, v) = uv(u^\iota + v^\iota)^{-1/\iota}$ so that $q(\theta) = (1 + \theta^\iota)^{1/\iota}$, which is bounded between 0 and 1. We set $\iota = 0.9$ by nonlinear least squares to match the empirical relationship between aggregate market tightness and job-finding rates. We measure empirical labor market tightness as job openings from the Job Openings and Labor Turnover Survey divided by household unemployment in the Current Population Survey (CPS). We measure empirical job-finding rates over the same time period that JOLTS data is available (2000-2019). We set the exogenous separation rate $s = 0.031$ to the average monthly separation rate from 1951 to 2019. Job-finding rates and separation rates are computed using data on short-term unemployment and employment from the CPS, following Shimer (2005). These values of β and s imply that the pass-through parameter $\psi = 1 - \beta(1-s)$ equals 0.034.

We now rewrite some of the key equations in our numerical model recursively, given the Markovian structure for productivity. Let $\pi(z'|z)$ denote the probability of aggregate productivity transitioning from z to z'. Let $\tilde{Y}(z; z_0)$ denote the EPDV of future output, conditional on initial productivity z_0, effort schedule $a(z; z_0)$, and current productivity z:

$$\tilde{Y}(z; z_0) = za(z; z_0) + \sum_{z' \in \mathcal{Z}} \beta(1-s)\tilde{Y}(z'; z_0))\pi(z'|z).$$

It follows that $Y(z_0) = \tilde{Y}(z_0; z_0)$. Recall that the optimal effort depends on z_0 through $w_{-1}(z_0)$, which is an equilibrium object in our model. Define the worker's expected present discounted utility from starting work at z_0, $\tilde{\mathcal{E}}(z_0)$, taking as given the effort schedule $a(\cdot; z_0)$ and the wage schedule $w(\cdot; z_0)$ defined in Proposition 6:

$$\begin{aligned}\tilde{\mathcal{E}}(z_0) = \frac{1}{\psi}\ln w_{-1}(z_0) + \mathbb{E}_z\Bigg[-\sum_{t=0}^{\infty}[\beta(1-s)]^t \frac{1}{\psi}\frac{1}{2}(\psi h'(a(z_t; z_0))\sigma_\eta)^2 - \\ \sum_{t=0}^{\infty}[\beta(1-s)]^t h(a(z_t; z_0)) + \sum_{t=0}^{\infty}[\beta(1-s)]^t \beta s \omega(z_{t+1})\Bigg],\end{aligned}$$

where

$$\omega(z) = \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t \ln b(z_t) \mid z_0 = z\right].$$

We can write the term in brackets recursively as $W(z_0; z_0)$, where

$$\begin{aligned}W(z; z_0) = -\frac{1}{\psi}\frac{1}{2}(\psi h'(a(z; z_0))\sigma_\eta)^2 - h(a(z; z_0)) + \sum_{z' \in \mathcal{Z}} \beta s \omega(z')\pi(z'|z_0) \\ + \sum_{z' \in \mathcal{Z}} \beta(1-s) W(z'; z_0)\pi(z'|z_0).\end{aligned}$$

B.3.1 Algorithm to solve for the optimal contract, given z_0

Fix an initial $z_0 \in \mathcal{Z}$. To solve for the optimal contract beginning at z_0, we perform a bisection search over the job-filling rate $w_{-1}(z_0)$. Let n index iterations over our guess of $w_{-1}(z_0)$. Then, for a given $w^n_{-1}(z_0)$, we can solve for the optimal effort schedule $a^n(z; z_0)$. With values of $a^n(z; z_0)$ and $w^n_{-1}(z_0)$, we can recursively solve for the EPDV of the utility offered by the contract $\mathcal{E}^n(z_0)$. We then check whether $\mathcal{E}^n(z_0) = \omega(z_0)$, and accordingly update the lower and upper bounds for w_{-1} in the next iteration, $\underline{w}^{n+1}_{-1}$ and $\bar{w}^{n+1}_{-1}$, respectively. We continue this process until convergence of $w_{-1}(z_0)$. We solve for the implied $\theta(z_0)$ from the free entry condition. Below, we describe the algorithm in further detail.

1. Set $n = 1$. Set $0 < \underline{w}^n < \bar{w}^n$.

2. Set $w^n(z_0) = \frac{1}{2}(\underline{w}^n + \bar{w}^n)$.

3. Solve numerically for effort as $a^n(z; z_0) = \tilde{a}(z, w^n_{-1}(z_0))$, where

$$\tilde{a}\left(z, w_{-1}\right) = \left[\frac{z\tilde{a}}{\psi\left(w_{-1}\right)} - \frac{\psi}{\epsilon}\left(h'\left(\tilde{a}\right)\sigma_\eta\right)^2\right]^{\frac{\epsilon}{1+\epsilon}}.$$ [37]

4. Set $j = 1$. Make initial guess for $W^{j,n}\left(z; z_0\right)$.

5. Update $W^{j+1,n}(\cdot; z_0)$ as

$$W^{j+1,n}\left(z; z_0\right) = -\frac{1}{\psi}\frac{1}{2}(\psi h'(a^n(z))\sigma_\eta)^2 - h(a^n(z)) + \sum_{z' \in \mathcal{Z}} \beta s \omega\left(z'\right)\pi\left(z'|z\right) + \sum_{z' \in \mathcal{Z}} \beta\left(1-s\right)W^{j,n}\left(z'; z_0\right)\pi\left(z'|z\right)$$

6. Repeat (8) until $\|W^{j,n+1}(\cdot; z_0) - W^{j,n}(\cdot; z_0)\| < \delta_2$ for some small tolerance $\delta_1 > 0$. Define $\mathcal{E}^n(z_0) = \frac{1}{\psi}\ln w^n_{-1}(z_0) + W^{j,n}(z_0; z_0)$.

7. If $\mathcal{E}^n(z_0) > \omega(z_0)$ then set $\bar{w}^{n+1} = w^n(z_0)$. If $\mathcal{E}^n(z_0) < \omega(z_0)$, then set $\underline{w}^{n+1} = w^n(z_0)$. Note that $\omega(z_0)$ can be computed by a simple value function iteration.

8. Repeat steps (2)-(7) until $|\mathcal{E}^n(z_0) - \omega(z_0)| < \delta_2$ or $|w^{n+1}_{-1}(z_0) - w^n_{-1}(z_0)| < \delta_2$ for some small tolerance $\delta_3 > 0$ to obtain $w_{-1}(z_0)$. Set $w_{-1} = w^n_{-1}$ and $a(z; z_0) = a^n(z; z_0)$.

9. Set $k = 1$. Make initial guess for $Y^k(z; z_0)$ for $z \in \mathcal{Z}$.

10. Update $Y^{k+1}(\cdot; z_0)$ as

$$Y^{k+1}\left(z; z_0\right) = za(z; z_0) + \sum_{z' \in \mathcal{Z}} \beta\left(1-s\right)Y^k\left(z'; z_0\right)\pi\left(z'|z\right).$$

11. Repeat (4) until $\|Y^{k+1}\left(\cdot; z_0\right) - Y^k\left(\cdot; z_0\right)\| < \delta_3$ for some small tolerance $\delta_3 > 0$. Set $Y(z_0) = Y^n(z_0; z_0)$.

12. Define $J^*(z_0) = Y(z_0) - w_{-1}(z_0)/\psi$. Define $J(z_0) = \max\{J^*(z_0), 0\}$. Define $q(z_0) = \min\{\kappa/J(z_0), 1\}$. Define $\theta(z_0) = q^{-1}(q^n(z_0))$, where $q(\theta) = \frac{1}{(1+\theta^\iota)^{1/\iota}}$.

[37] We restrict attention to positive solutions of this implicit equation. We verify uniqueness of the obtained solution at our estimated parameters.

We repeat this procedure for all values of $z_0 \in \mathcal{Z}$ to obtain the equilbrium objects $\theta(z_0)$, $Y(z_0)$, $w_{-1}(z_0)$, and $a(z; z_0)$.

B.3.2 Details on Simulation

Our set of targeted moments includes two moments that depend on within-contract, idiosyncratic realizations: the standard deviation of annual (YoY) wage growth ($\text{std}(\Delta \ln w_{it})$) and the average pass-through from idiosyncratic shocks to firm profits to wages ($\mathbb{E}[^{\partial \ln w_{it}}/_{\partial \ln y_{it}}]$), and two moments which can be computed from aggregate time series simulated in the model: the cyclicality of new hire wages ($^{\partial \mathbb{E}[\ln w_0]}/_{\partial u}$) and average unemployment ($\bar{u}_t$). To compute these moments for a given set of parameters $\Omega := \{\epsilon, \sigma_\eta, \chi, \gamma\}$, we solve the model for each initial $z_0 \in \mathcal{Z}$ following the procedure outlined in Section B.3.1 to obtain $a(\cdot|z_0)$, $w_{-1}(z_0)$, and $\theta(z_0)$. We then simulate the economy with aggregate shocks and compute moments.

Simulating $\text{std}(\Delta \ln w_{it})$ and $\mathbb{E}[^{\partial \ln w_{it}}/_{\partial \ln y_{it}}]$. We simulate a panel of $I = 10,000$ idiosyncratic η_{it} shocks of length $T = 828 + 500$ (and one sequence of aggregate z_t shocks of length T). For each period t and worker i, we simulate separations and job-finding shocks consistent with the exogenous probability of separation s and endogenous job-finding probability $\phi(\theta(z_t))$. All workers are employed at the beginning of $t = 0$. During job spells and given realizations of z_t and η_{it}, we can compute log wages and individual pass-through according to the equations derived in Section B.1. For job spells that last at least 13 months, we can compute YoY log wage growth as $\ln w_{i,t+12} - \ln w_{it}$ (for each year of employment). We can compute the pass-through from idiosyncratic output shocks to the wages $\partial \ln w_{it}/\partial \ln y_{it}$ as $\psi h'(a_{it})(a_{it} + \eta_{it})$. We discard the first $t_{\text{burn-in}} = 500$ periods as a burn-in period. We then compute the pooled variance of YoY log wage growth and the average pass-through across all periods of employment for $t \geq t_{\text{burn-in}}$.[38] Our simulation procedure captures composition effects of initial z_0 on the employment contracts.

Simulating $^{d\mathbb{E}[\ln w_0]}/_{du}$ and $\bar{u}_t$. We simulate 10,000 z_t sequences of length $T = 828$ periods (with an additional burn-in period of length 500 periods). For each z_t path, we compute the path for unemployment as

$$u_{t+1} = u_t + s(1 - u_t) - \phi(\theta(z_t))u_t(1 - s)$$

[38] We condition on positive output when computing the average pass-through, since individual output in our model can be negative.

given initial condition $u_0 = 0.06$. The expected log wage of new hire wages in our model is

$$\mathbb{E}_{\eta_{it}}[\ln w_0(z_t)] = \ln w_{-1}(z_t) - \frac{1}{2}(\psi h'(a(z_t|z_t))\sigma_\eta)^2.$$

We compute $\bar{u}_t$ as the average unemployment u_t for $t \geq t_{\text{burn-in}}$. We measure ${}^{d\mathbb{E}[\ln w_0]}/_{du}$ in the model by running an OLS regression of $\mathbb{E}[\ln w_0](z_t)$ on u_t and a constant in the simulated data for $t \geq t_{\text{burn-in}}$. We report cross-simulation averages for both moments.

Simulating ρ_p and σ_p We simulate a panel of $I = 10,000$ workers of length $T = 828 + 500$ (and 25 sequences of aggregate z_t shocks of length T). For each period t and worker i, we simulate separations and job-finding shocks consistent with the exogenous probability of separation s and endogenous job-finding probability $\phi(\theta(z_t))$. We compute average labor productivity as the average output among employed workers within a quarter. We report averages of the unconditional standard deviation and autocorrelation of average labor productivity across the 25 simulations. This simulation procedure captures composition effects of initial z_0 on the employment contracts.

B.3.3 Estimation Algorithm

We implement a version of the Tik-Tak algorithm, a multi-start global optimization algorithm, as described by Arnoud et al. (2019), to minimize the following objective function

$$J(\Omega) = (\tilde{m}(\Omega) - m)' W(m)(\tilde{m}(\Omega) - m),$$

where Ω is a vector of the parameters to be estimated, $\tilde{m}(\Omega)$ is a vector of the targeted moments computed using the model simulated data given the parameter vector Ω, and m is the vector of targeted empirical moments. We set the weight matrix W to $W_{j,j} = 1/m_j^2$ for each targeted moment j (and 0, otherwise). Thus, the objective function to minimize is the sum of squared percentage differences between simulated and empirical moments to account for differences in scale between the targeted moments. We have experimented with different derivative-free local optimization algorithms, such as BOBYQA and the Nelder-Mead Simplex Algorithm, for the local optimization step. All estimation results correspond to solutions obtained using a combination of the Nelder-Mead Simplex Algorithm and BOBYQA algorithm with 1,000 initial points. We implement a pre-testing stage to detect promising regions of the parameter space by evaluating the objective function at a large number of initial points drawn from Sobol sequences; we use the 1,000 points that yield the lowest values of the objective function as the initial points in the global search iterations.

Technical detail on computing the new hire wage cyclicality For some parameter values in the calibration, $q(\theta(z_0))$ may hit its upper bound of 1. When $q(\theta(z_0)) = 1$, the implied new hire wage that we compute for z_0 would thus not be an observed wage, as $\phi(\theta(z_0)) = 0$. The other wage moments do not pose this challenge because that we simulate employment spells in accordance with the endogenous job-finding probabilities.

To address this discrepancy with the data, we penalize parameters for which $q(\theta(z_0)) = 1$ for values of $\ln z_0$ within three unconditional standard deviations of μ_z and truncate the simulation to fall within this range.[39] We do not penalize violations for lower values of z_0 as the probability of reaching them is low, and it is also reasonable to expect that the $q(\theta(z_0)) \approx 1$ for some very low z_0. We have explored alternative approaches, with the results being largely consistent across different approaches.

B.3.4 Calculating Incentive Wage Cyclicality

Non-incentive wage cyclicality reflects movements in the promised utility of workers. For a given calibration, we calculate how the value of a filled job moves with exogenous productivity $dJ(z_0)/dz_0$. The "direct effect" of z_0 on the expected present discounted value of profits per worker, given the AR(1) process for $\ln z$, can be approximated as

$$\frac{dJ(z_0)}{dz_0}^{Direct} = \sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\mathbb{E}_0[a^*(z_t)\rho^t z_t]}{z_0}.$$

That is, the direct effect is the effect that z has on profits holding fixed the optimally contracted choice of effort and wages. We calculate NWC as

$$\text{NWC}(z_0) = \frac{dJ(z_0)}{dz_0} - \frac{dJ(z_0)}{dz_0}^{Direct}.$$

The share of wage fluctuations attributable to incentives is then the negative of one minus $\text{NWC}(z_0)$ divided by the cyclicality of the expected present discounted value of wage payments $^{d\mathcal{W}^*(z_0)}/_{dz_0}$.

B.3.5 Construction of Impulse Responses

We compute the impulse response to a one (conditional) standard deviation (σ_z) shock to $\ln z_0$ in an economy that is at an aggregate non-stochastic steady state. We construct nonlinear perfect foresight impulse responses to a one-time shock to productivity at time 0

[39] We still solve the model on a wider grid to obtain a more precise solution.

that decays at rate ρ_z. We define the non-stochastic steady state of $\log z$ to be 0, dropping the normalization of μ_z that ensures $\mathbb{E}[z_t] = 1$ given that $\mu_z \approx 0$.

We first solve for the non-stochastic steady state of the model, where $z_t = z_{ss} = 1$, $\theta_t = \theta(z_{ss})$, and $u_t = u_{ss} = \frac{s}{s+\phi(\theta(z_{ss}))(1-s)}$ for all t. We next solve for the path of $\theta_t(\{z_s\}_{s\geq t})$, given a sequence of shocks $\{z_t\}$. Finally, we solve for the path of unemployment u_t, given the path of θ_t, setting $u_0 = u_{ss}$.[40] We construct these impulse responses in a finite horizon contract setting and set the length of the contract, T, to be 240 model periods (20 years), which is a close approximation to the infinite horizon contract environment.[41]

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[40] The calibration was done for the infinite horizon contract environment and targeted the stochastic mean of unemployment rather than steady state unemployment rate as implied in a non-stochastic model. Therefore, the steady state across the models need not be the same, although they are very close in practice.

[41] There is an additional term in the law of motion for unemployment in the finite horizon contract setting because workers separate with probability one after they have completed their contract without experiencing a separation shock. However, the measure of workers that do not separate by time T is essentially zero, given that $T = 240$. Therefore, we ignore this inflow into unemployment in this numerical exercise.

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